YOUNG DUALITY OF VARIATIONAL INEQUALITIES. AN APPLICATION FOR THE ANALYSIS OF INTERACTIONS IN PRODUCTION NETWORKS

N. K. Obrosova, A. A. Shananin

We develop a mathematical technique of Young dual variational inequalities, which are used to model market equilibrium in a network of production clusters that are heterogeneous from a technological point of view. Two formulations of the problem are considered: for a closed system with a given constraint on resources and for an open system in which resources can be supplied from outside at given prices. A theorem is proved on the existence of a solution to the variational inequality corresponding to market equilibrium in an open system.

Keywords: production network, heterogeneity, variational inequality, Young duality, market equilibrium, resource allocation problem.

REFERENCES

1. Barro R.J., Sala-i-Martin X. *Economic growth*. Cambridge, London, MIT Press., 2004.

2. Chamberlin E.H. *The theory of monopolistic competition. Harvard economic studies*, 1969, Cambridge, Harvard Univ. Press. Translated to Russian under the title: Teoriya monopolisticheskoi konkurentsii (Reorientatsiya teorii stoimosti), Moscow: Ekonomika, 1996, 351 p.

3. Acemoglu D., Ozdaglar A., Tahbaz-Salehi A. The network origins of aggregate fluctuations. *Econometrica*, 2012, vol. 80, no. 5, pp. 1977–2016. doi: 10.3982/ECTA9623

4. Acemoglu D., Ozdaglar A., Tahbaz-Salehi A. Networks, shocks, and systemic risk. In: *The Oxford Handbook of the Economics of Networks*, eds. Yann Bramoulé et al, NY: Oxford Univ. Press, 2016, pp. 569–607. doi: 10.3386/w20931

5. Sarrazin T. *Europa braucht den Euro nicht*. München: Deutsche Verlags-Anstalt (DVA), 2012. Translated to Russian under the title: Evrope ne nuzhen evro, Moscow: AST Publ., 2015, 512 p.

6. Chevènement J.-P. *1914–2014 : l'Europe sortie de l'histoire?*, 2013, Editions Fayard, 350 p. Translated to Russian under the title: 1914–2014: Evropa vykhodit iz istorii, Moscow: AST Publ., 2015, 352 p.

7. Shananin A. Young duality and aggregation of balances. *Dokl. Math.*, 2020, vol. 102, no. 1, pp. 330–333. doi: 10.1134/S1064562420040171

8. Shananin A. Problem of aggregating of an input-output model and duality. *Comput. Math. and Math. Phys.*, 2021, vol. 61, no. 1, pp. 153–166. doi: 10.1134/S0965542521010085

9. Boranbayev S., Obrosova N., Shananin A. Production network centrality in connection to economic development by the case of Kazakhstan statistics. In: *Optimization and Applications: 12th Internat. Conf. (OPTIMA 2021): Proc.*, eds. Nicholas N. Olenev et al., 2021, Ser. Lecture Notes in Computer Science, vol. 13078, pp. 321–335. doi: 10.1007/978-3-030-91059-4_23

10. Obrosova N., Shananin A., Spiridonov A. On the comparison of two approaches to intersectoral balance analysis. *J. Physics: Conf. Ser.*, 2021, vol. 2131, no. 2. doi: 10.1088/1742-6596/2131/2/022112

11. Rassokha A., Shananin A. Inverse problems of the analysis of input-output balances. *Math. Models and Computer Simulations*, 2021, vol. 13, no. 6, pp. 943–954. doi: 10.1134/S2070048221060193

12. Kerimkhulle S., Obrosova N., Shananin A., Azieva G. The nonlinear model of intersectoral linkages of Kazakhstan for macroeconomic decision-making processes in sustainable supply chain management. *Sustainability*, 2022, vol. 14, no. 21. doi: 10.3390/su142114375

13. Boranbayev A., Obrosova N., Shananin A. Nonlinear input-output balance and Young duality: Analysis of Covid-19 macroeconomic impact on Kazakhstan. *Sib. Electron. Math. Reports*, 2022, vol. 19, no. 2, pp. 835–851. doi: 10.33048/semi.2022.19.070

14. Obrosova N., Shananin A., Spiridonov A. Nonlinear input-output model with nested CES technologies for the analysis of macroeconomic effects of a foreign trade shock. *Lobachevskii J. Math.*, 2023, vol. 4, no. 1, pp. 401–417. doi: 10.1134/S1995080223010304

15. Leontief W.W. *The Structure of American economy, 1919–1939: An empirical application of equilibrium analysis.* Oxford: Oxford Univ. Press, 1951, 264 p.

16. Ashmanov S.A. *Vvedenie v matematicheskuyu ekonomiku* [Introduction to mathematical economics]. Moscow: Nauka Publ., 1984, 293 p.

17. Acemoglu D., Akcigit U., Kerr W. Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual*, 2016, vol. 30, no. 1, pp. 273–335.
Available on: http://nrs.harvard.edu/urn-3:HUL.InstRepos:17527693 .

18. Barauskaite Kristina, Nguyen Anh D.M. Global intersectoral production network and aggregate fluctuations. *Economic Modelling*, 2021, vol. 102(C). doi: 10.1016/j.econmod.2021.105577

19. Acemoglu D., Azar P.D. Endogenous production networks. *Econometrica*, 2020, vol. 88, no. 1, pp. 33–82. doi: 10.3982/ECTA15899

20. Acemoglu D., Ozdaglar A., Tahbaz-Salehi A. Microeconomic origins of macroeconomic tail risks. *Am. Econ. Rev.*, 2017, vol. 107, no. 1, pp. 54–108. doi: 10.1257/aer.20151086

21. Acemoglu D., Ozdaglar A., Tahbaz-Salehi A. Systemic risk and stability in financial networks. *Am. Econ. Rev.*, 2015, vol. 105, no. 2, pp. 564–608. doi: 10.1257/aer.20130456

22. Baqaee D.R. Cascading failures in production networks. *Econometrica*, 2018, vol. 86, no. 5, pp. 1819–1838. doi: 10.3982/ECTA15280

23. Aubin J.P. *L'analyse Non Linéaire et ses Motivations Economiques.* Paris: Masson, 1984, 214 p. Translated to Russian under the title: Nelineinyi analiz i ego ekonomicheskie prilozheniya. Moscow: Mir Publ., 1988, 264 p.

24. Nikaido H. *Convex structures and economic theory.* NY: Acad. Press, 1968, 405 p. ISBN: 9781483230030. Translated to Russian under the title: Vypuklye struktury i matematicheskaya ekonomika, Moscow: Mir Publ., 1972, 520 p.

25. Shananin A.A. Duality for generalized programming problems, and variational principles in models of economic equilibrium. *Doklady Akademii Nauk*, 1999, vol. 366, no. 4, pp. 462–464 (in Russian).

26. Shananin A.A. Integrability problem and the generalized nonparametric method for the consumer demand analysis. In: *Proceedings of Moscow Inst. Phys. Technol.*, 2009, vol. 4, no. 1, pp. 84–98 (in Russian).

27. Miller R.E., Blair P.D. *Input-output analysis: Foundations and extensions.* 2nd ed, Cambridge: Cambridge Univ. Press, 2009, pp. 1–750.

28. Wixted B., Yamano N., Webb C. Input-output analysis in an increasingly globalised world: Applications of OECDs harmonised international tables. In: *OECD Science, Technology and Industry: working papers*, 2006, no. 2006/07, Paris: OECD Publ., 2006. doi: 10.1787/303252313764

29. O'Mahony Mary, Timmer Marcel P. Output, input and productivity measures at the industry level: The EU KLEMS database. *Economic J.*, 2009, vol. 119, no. 538, pp. F374–F403.
doi 10.1111/j.1468-0297.2009.02280.x

30. Timmer M.P. [et al.] An Illustrated user guide to the world input-output database: The Case of global automotive production. *Review of International Economics*, 2015, vol. 23, pp. 575–605. doi 10.1111/roie.12178

Nataliia Obrosova, Cand. Sci. (Phys.-Math.), Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, Moscow, 119333 Russia, e-mail: nobrosova@ya.ru .

Alexander Shananin, Dr. Phys.-Math. Sci., Prof., RAS Academician, Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, Moscow, 119333 Russia, e-mail: alexshan@ya.ru .

Cite this article as: N. K. Obrosova, A. A. Shananin. Young duality of variational inequalities. An

application for the analysis of interactions in production networks. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 3, pp. 88–105.