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A BICOMPOSITION OF CONICAL PROJECTIONS

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We consider a decompositional approach to the problem of finding the orthogonal projection of a given point onto a convex polyhedral cone represented by a finite set of its generators. The reducibility of an arbitrary linear optimization problem to such projection problem potentially makes this approach one of the possible new ways to solve large-scale linear programming problems. Such an approach can be based on the idea of a recurrent binary decomposition that splits the original large-scale problem into a binary tree of conical projections corresponding to a sequential decomposition of the initial cone into the sum of lesser subcones. The key operation of this approach is solving the problem of projecting of a certain point onto a cone represented as the sum of two subcones with the smallest possible modification of these subcones and their arbitrary choice. Three iterative algorithms implementing this basic operation are proposed, their convergence is proved, and numerical experiments demonstrating both the computational efficiency of the algorithms and certain problems of their application are performed.

Keywords: orthogonal projection, polyhedral cones, decomposition, linear optimization.

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