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**THE TIME-OPTIMAL CONTROL PROBLEM OF SEQUENTIAL TRAVERSAL  
OF SEVERAL POINTS BY A DUBINS CAR****A. M. Mayer, A. A. Galyaev**

A time-optimal control problem of sequential traversal of three target points on the plane by a Dubins car is considered. The Dubins car model is used to describe the motion of an object in a horizontal plane with a constant speed and limited maneuverability. Fixed and unfixed sequences of traversal of target points are considered. The problem is discrete-continuous and contains three target sets. The difficulty of finding a solution lies in the impossibility to divide the problem into a series of tasks with two target points since it is necessary to consider information about all target points to minimize the traversal time. Necessary optimality conditions are formulated and used to develop an algorithm for constructing an optimal trajectory in the far zone. An explicit form of an optimal program control is obtained, and the problem of optimal control synthesis is solved. For a problem with a fixed traversal sequence, an algorithm for constructing an optimal trajectory for visiting three and two target points is developed. The results of the two algorithms are compared. The most interesting results of trajectory modeling for various cases of mutual position of target points are presented graphically. For a problem with an unfixed traversal sequence, a solution algorithm is constructed and the boundaries of the regions where the traversal sequence changes are found.

Keywords: Dubins car, time-optimal control problem, optimal trajectory, fixed targets, target traversal algorithm.

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