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CLOSED MAPPINGS AND CONSTRUCTION OF EXTENSION MODELS

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The problem of reachability in a topological space is studied under constraints of asymptotic nature arising from weakening the requirement that the image of a solution belong to a given set. The attraction set that arises in this case in the topological space is a regularization of certain kind for the image of the inverse image of the mentioned set (the image and the inverse image are defined for generally different mappings). When constructing natural compact extensions of the reachability problem with constraints of asymptotic nature generated by a family of neighborhoods of a fixed set, the case was studied earlier where the topological space in which the results of one or another choice of solution are realized satisfies the T_2 axiom. In the present paper, for a number of statements related to compact extensions, it is possible to use for this purpose the T_1 -space, which seems to be quite important from a theoretical point of view, since it is possible to find out exactly what is the role of the T_2 axiom in questions related to correct extensions of reachability problems. We study models of extensions using ultrafilters of a broadly understood measurable space with detailing of the main elements in the case of a reachability problem in the space of functionals with the topology of the Tikhonov power of the real line with the usual $|\cdot|$ -topology. The general constructions of extension models are illustrated by an example of a nonlinear control problem with phase constraints.

Keywords: attraction set, extension model, ultrafilter.

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