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FEJÉR-TYPE ITERATIVE PROCESSES IN THE CONSTRAINED QUADRATIC MINIMIZATION PROBLEM

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The paper presents an overview of methods for solving an ill-posed problem of constrained convex quadratic minimization based on the Fejér-type iterative methods, which widely use the ideas and approaches developed in the works of I. I. Eremin, the founder of the Ural research school of mathematical programming. Along with a problem statement of general form, we consider variants of the original problem with constraints in the form of systems of equalities and inequalities, which have numerous applications. In addition, particular formulations of the problem are investigated, including the problem of finding a metric projection and a linear programming problem, which are of independent interest. A distinctive feature of these methods is that not only convergence but also stability with respect to errors in the input data are established for them; i. e., the methods generate regularizing algorithms in contrast to the direct methods, which do not have this property.

Keywords: quadratic minimization, ill-posed problem, linear constraints, Fejér process, regularizing algorithm.

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