# OPTIMIZATION OF THE OPTIMAL VALUE FUNCTION IN PROBLEMS OF CONVEX PARAMETRIC PROGRAMMING

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We consider a problem of convex parametric programming in which the objective function and the constraint functions are convex functions of an outer parameter. Computational procedures are suggested for finding the maximal and minimal values of the optimal value function and for finding inner and outer approximations of the set of parameters for which the problem is consistent. All procedures are based on the application of support functions. Illustrative examples are provided.

Keywords: parametric optimization, optimal value function, support function, inner and outer approximation.

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