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THE STRUCTURE OF THE ESSENTIAL SPECTRUM AND THE DISCRETE SPECTRUM OF THE ENERGY OPERATOR FOR SIX-ELECTRON SYSTEMS IN THE HUBBARD MODEL. THE SECOND SINGLET STATE

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We consider the energy operator of six-electron systems in the Hubbard model and study the structure of the essential spectrum and the discrete spectrum of the system for the second singlet state of the system. In the one- and two-dimensional cases, it is shown that the essential spectrum of the six-electron second singlet state operator consists of unions of seven segments, and the discrete spectrum of the system consists of a single eigenvalue lying below (above) the domain of the lower (upper, respectively) edge of the essential spectrum of this operator. In the three-dimensional case, there are the following situations: (a) the essential spectrum of the six-electron second singlet state operator consists of unions of seven segments, and the discrete spectrum of this operator consists of a single eigenvalue; (b) the essential spectrum of the six-electron second singlet state operator consists of unions of four segments, and the discrete spectrum of this operator is empty; (c) the essential spectrum of the six-electron second singlet state operator consists of unions of two segments, and the discrete spectrum of this operator is empty; (d) the essential spectrum of the six-electron second singlet state operator consists of a single segment, and the discrete spectrum of this operator is empty. Conditions are found under which each of the situations takes place.

Keywords: Hubbard model of six-electron systems, spectrum, essential spectrum, discrete spectrum, octet state, quintet state, triplet state, singlet state.

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