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## MINIMIZING SEQUENCES IN A CONSTRAINED DC OPTIMIZATION PROBLEM

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A smooth nonconvex optimization problem is considered, where the equality and inequality constraints and the objective function are given by DC functions. First, the original problem is reduced to an unconstrained problem with the help of I. I. Eremin's exact penalty theory, and the objective function of the penalized problem also turns out to be a DC function. Necessary and sufficient conditions for minimizing sequences of the penalized problem are proved. On this basis, a "theoretical method" for constructing a minimizing sequence in a penalized problem with a fixed penalty parameter is proposed and the convergence of the method is proved. The well-known local search method and its properties are used for developing a new global search scheme based on global optimality conditions with varying penalty parameter. The sequence constructed using the global search scheme turns out to be minimizing in the "limit" penalized problem, and each of its terms  $z^{k+1}$  turns out to be an approximately critical vector for the local search method and an approximate solution of the current penalized problem  $(\mathcal{P}_k) \triangleq (\mathcal{P}_{\sigma_k})$ . Finally, under an additional condition of "approximate feasibility", the constructed sequence turns out to be minimizing for the original problem with DC constraints.

Keywords: DC function, exact penalty, linearized problem, minimizing sequence, global optimality conditions, local search, global search, critical vector, resolving approximation.

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