

MSC: 47N05, 37N25, 37N40

DOI: 10.21538/0134-4889-2023-29-3-168-184

ON THE OPTIMAL CORRECTION OF IMPROPER CONVEX PROGRAMMING PROBLEMS BASED ON THE METHOD OF QUASI-SOLUTIONS

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The work continues the author's research on the construction of possible approximations for improper problems of convex programming. The problem of minimizing the objective function of the original problem on the set of minimum points of the Chebyshev norm of the constraint discrepancy is defined as a basic model for correcting an improper problem. For this setting, one of the classical methods of regularization of ill-posed extremal problems is used, namely, the method of quasi-solutions. This method is based on the transition to a problem of unconstrained minimization by aggregation of the constraint function of the original problem. For this purpose, a modification of the penalty function method is used, namely, the generalized inverse barrier function method. This approach seems to be promising from the point of view of the numerical implementation of the quasi-solution method. Convergence conditions are formulated for the proposed method, including the case where the input data are given inaccurately. Special attention is paid to finding the value of optimal correction of the constraints in the improper problem of convex programming under study and to calculating the optimal value of the regularization parameter in the method of quasi-solutions.

Keywords: convex programming, improper problem, optimal correction, method of quasi-solutions, barrier function methods.

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Received March 27, 2023

Revised July 14, 2023

Accepted July 21, 2023

Funding Agency: This study is a part of the research carried out at the Ural Mathematical Center and supported by the Ministry of Science and Higher Education of the Russian Federation (agreement no. 075-02-2023-913).

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Cite this article as: V. D. Skarin. The method of quasi-solutions based on barrier functions in the analysis of improper convex programming problems. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 3, pp. 168–184 .