COMPARISON AND POLYHEDRAL PROPERTIES OF VALID INEQUALITIES FOR A POLYTOPE OF SCHEDULES FOR SERVICING IDENTICAL REQUESTS

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The paper considers the convex hull of a set of schedules for servicing identical requests by parallel devices. Precedence conditions are given on the set of requests. All requests enter the service queue simultaneously and have the same service duration. Interruptions in request servicing are prohibited. Time is discrete. The polyhedral properties of some previously constructed classes of valid inequalities are studied. The "depth" cuts are compared, and the strongest subclasses of cuts are found. The mutual arrangement of the schedule polyhedron and hyperplanes generated by inequalities is also studied.

Keywords: schedules, polytope, valid inequality, comparison of inequalities.

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