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POWER DEGREES IN DYNAMIC MULTI-AGENT SYSTEMS¹

Leon Petrosyan, David Yeung, and Yaroslavna Pankratova

Dynamic multi-agent systems connected in network are considered. To define the power of each agent the analogue of characteristic function is introduced. The values of this characteristic function for each coalition (subset of agents) are calculated as joint payoff of players from this coalition plus payoffs (multiplied on some discount factor) of players which do not belong to the coalition S but have connections with players from S . We suppose that the dynamic of the system is prescribed (this maybe cooperation, Nash equilibrium or any other behaviour). Thus, the characteristic function is evaluated along the prescribed trajectory of agents. And it measures the worth of coalitions under the motion along this trajectory instead of under minimax confrontation or the Nash non-cooperative stance. As solution we consider the proportional solution and introduce Power degrees of an agent based on proportional solution. It is shown that the Power degree (PD) belongs to the Core. PD rank agents according to their importance.

Keywords: multi-agent system and proportional solution and power degree.

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Leon Aganesovich Petrosyan, Dr. Phys.-Math. Sci., Prof., Saint Petersburg State University, St. Petersburg, 199034 Russia, e-mail: l.petrosyan@spbu.ru.

David W.K. Yeung, Prof. Dr. Dr.h.c., Hong Kong Shue Yan University, Hong Kong, China, e-mail: dwkyeung@hkysyu.edu.

Yaroslava Borisovna Pankratova, Cand. Sci. (Phys.-Math.), Saint Petersburg University, St. Petersburg, 199034 Russia, e-mail: y.pankratova@spbu.ru.

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