

## ON INTERSECTIONS OF NILPOTENT SUBGROUPS IN FINITE GROUPS WITH SIMPLE SOCLE FROM THE “ATLAS OF FINITE GROUPS”

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Earlier, the author described up to conjugation all pairs  $(A, B)$  of nilpotent subgroups of a finite group  $G$  with socle  $L_2(q)$  for which  $A \cap B^g \neq 1$  for any element  $g$  of  $G$ . A similar description was obtained by the author later for primary subgroups  $A$  and  $B$  of a finite group  $G$  with socle  $L_n(2^m)$ . In this paper, we describe up to conjugation all pairs  $(A, B)$  of nilpotent subgroups of a finite group  $G$  with simple socle from the “Atlas of Finite Groups” for which  $A \cap B^g \neq 1$  for any element  $g$  of  $G$ . The results obtained in the considered cases confirm the hypothesis (Problem 15.40 from the “Kourovka Notebook”) that a finite simple non-Abelian group  $G$  for any nilpotent subgroups  $N$  contains an element  $g$  such that  $N \cap N^g = 1$ . Keywords: finite group, nilpotent subgroup, intersection of subgroups, Fitting subgroup.

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