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**ASYMPTOTICS OF A SOLUTION TO AN OPTIMAL CONTROL PROBLEM
WITH A TERMINAL CONVEX PERFORMANCE INDEX
AND A PERTURBATION OF THE INITIAL DATA**

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In this paper, we investigate a problem of optimal control over a finite time interval for a linear system with constant coefficients and a small parameter in the initial data in the class of piecewise continuous controls with smooth geometric constraints. We consider a terminal convex performance index. We substantiate the limit relations as the small parameter tends to zero for the optimal value of the performance index and for the vector determining the optimal control in the problem. We show that the asymptotics of the solution can be of complicated nature. In particular, it may have no expansion in the Poincare sense in any asymptotic sequence of rational functions of the small parameter or its logarithms.

Keywords: optimal control, terminal convex performance index, asymptotic expansion, small parameter.

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