Trudy Instituta Matematiki i Mekhaniki UrO RAN

Vol. 29 No. 2 2023

MSC: 42A10, 42B35, 65T60

DOI: 10.21538/0134-4889-2023-29-2-287-293

RECONSTRUCTION OF A FUNCTION ANALYTIC IN A DISK FROM THE BOUNDARY VALUES OF ITS REAL PART USING INTERPOLATION WAVELETS

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For a function $f(z)$ analytic in a disk, a method of approximate reconstruction from known (or arbitrarily specified) boundary values of its real part (under the condition of its continuity) using interpolation wavelets is proposed; the method is easy to implement numerically. Despite the fact that there are known exact analytical formulas for solving this problem, the explicit formulas for approximating the function $f(z)$ proposed here are much easier to apply in practice, since the previously known exact formulas lead to the necessity to apply numerical integration methods when calculating convolutions of functions with Poisson or Schwartz kernels. For the approximations used in this paper, effective upper bounds are obtained for the error of approximation of functions analytic in the disk by interpolation wavelets in the spaces $L_p(0, 2\pi)$ for any $p \geq 2$. These estimates can be used to find the parameters of the wavelets from a desired accuracy of recovering the function $f(z)$. Note that if the real part of $f(z)$ is continuous on the boundary of the disk, then the continuity of $f(z)$ in the closure of the disk cannot be guaranteed; that is why it is impossible to estimate the approximation error for $f(z)$ in the uniform metric (for $p = \infty$) in the general case.

Keywords: multiresolution approximation, scaling function, interpolation wavelets, trigonometric polynomials, approximation order, function approximation.

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Received April 3, 2023, Revised April 24, 2023
Accepted May 15, 2023

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Cite this article as: N. I. Chernykh. Reconstruction of a function analytic in a disk from the boundary values of its real part using interpolation wavelets. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 2, pp. 287–293 .