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## SOME PROPERTIES OF ULTRAFILTERS RELATED TO THEIR USE AS GENERALIZED ELEMENTS

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Ultrafilters of widely understood measurable spaces and their application as generalized elements in abstract reachability problems with constraints of asymptotic nature are considered. Constructions for the embedding of conventional solutions, which are points of a fixed set, into the space of ultrafilters and representations of “limit” ultrafilters realized with topologies of the Wallman and Stone types are studied. The structure of the attraction set is established using constraints of asymptotic nature in the form of a nonempty family of sets in the space of ordinary solutions. The questions of implementation up to any preselected neighborhood of the attraction sets in the topologies of the Wallman and Stone types are studied. Some analogs of the mentioned properties are considered for the space of maximal linked systems.

Keywords: attraction set, constraints of asymptotic nature, ultrafilter.

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