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FREE VIBRATION ANALYSIS OF A CYLINDRICAL SHELL OF VARIABLE THICKNESS PARTIALLY FILLED WITH A FLUID

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The paper investigates the natural vibration frequencies of circular cylindrical shells of rotation, completely or partially filled with an ideal compressible fluid. The thickness of the shells is not constant and varies in the meridional direction according to different laws. The behavior of the elastic structure and compressible fluid is described within the framework of the classical shell theory using the Euler equations. The effects of sloshing on the free surface of the fluid are not considered. The equations of motion of the shell together with the corresponding geometric and physical relations are reduced to a system of ordinary differential equations in new unknowns. The acoustic wave equation is transformed to a system of ordinary differential equations by applying the generalized differential quadrature method. The solution to the formulated boundary value problem is found using Godunov's orthogonal sweep method. To calculate the natural frequencies of vibration, a stepwise procedure is used in combination with the refinement by the half-division method. The reliability of the obtained results is verified by comparing them with known numerical solutions. The behavior of minimum vibration frequencies at stepwise (linear and quadratic, having symmetric and asymmetric forms) and harmonic (with positive and negative curvature) variations in thickness is investigated for shells with different combinations of boundary conditions (simple support, rigid clamping and cantilever support) and levels of fluid filling. The study revealed the existence of configurations that provide at similar levels of filling a significant increase in the frequency spectrum compared to shells of constant thickness with the same weight constraints.

Keywords: classical shell theory, cylindrical shell, compressible fluid, Godunov's orthogonal sweep method, generalized differential quadrature method, free vibrations, variable thickness.

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