

MSC: 35R11, 34G20, 34A08

DOI: 10.21538/0134-4889-2023-29-2-248-259

QUASILINEAR EQUATIONS WITH A SECTORIAL SET OF OPERATORS AT GERASIMOV–CAPUTO DERIVATIVES

V. E. Fedorov, K. V. Boyko

The issues of unique solvability of the Cauchy problem are studied for a quasilinear equation solved with respect to the highest fractional Gerasimov–Caputo derivative in a Banach space with closed operators from the class $A_{\alpha, G}^n$ in the linear part and with a nonlinear operator continuous in the graph norm. A theorem on the local existence and uniqueness of a solution to the Cauchy problem is proved in the case of a locally Lipschitz nonlinear operator. Under the nonlocal Lipschitz condition for the nonlinear operator, the existence of a unique solution on a predetermined interval is shown. Abstract results are illustrated by examples of initial–boundary value problems for partial differential equations with Gerasimov–Caputo time derivatives.

Keywords: Gerasimov–Caputo fractional derivative, Cauchy problem, sectorial set of operators, resolving family of operators, quasilinear equation, local solution, nonlocal solution, initial–boundary value problem.

REFERENCES

1. Uchaikin V.V. *Metod drobnnykh proizvodnykh* [Method of fractional derivatives], Ulyanovsk, ArteShock Publ., 2008, 512 p. ISBN: 978-5-904198-01-5.
2. Tarasov V.E. *Fractional dynamics: Applications of fractional calculus to dynamics of particles, fields and media*, NY, Springer, 2011, 505 p. ISBN 978-3-642-14003-7.
3. Samko S.G., Kilbas A.A., Marichev O.I. *Fractional integrals and derivatives. Theory and applications*. Philadelphia, Gordon and Breach Science Publ., 1993, 976 p. ISBN: 9782881248641. Original Russian text was published in Samko S.G., Kilbas A.A., Marichev O.I. *Integraly i proizvodnye drobnogo poryadka i nekotorye ikh prilozheniya*, Minsk, Nauka i Tekhnika Publ., 1987, 638 p.
4. Prüss J. *Evolutionary integral equations and applications*, Basel, Springer, 1993, 366 p. doi: 10.1007/978-3-0348-8570-6
5. Podlubny I. *Fractional differential equations*, San Diego, Boston, Academic Press, 1999, 340 p. ISBN: 9780080531984.
6. Pskhu A.V. *Urvneniya v chastnykh proizvodnykh drobnogo poryadka* [Equations in partial derivatives of fractional order], Moscow, Nauka Publ., 2005, 199 p. ISBN: 5-02-033721-8.
7. Kilbas A.A., Srivastava H.M., Trujillo J.J. *Theory and applications of fractional differential equations*, Amsterdam, Elsevier Science Publ., 2006, 540 p. ISBN: 978-0-444-51832-3.
8. Kostić M. *Abstract Volterra integro-differential equations*, Boca Raton, CRC Press, 2015, 484 p. doi: 10.1201/b18463
9. Fedorov V.E., Boyko K.V., Fuong T.D. Initial problems for some classes of linear evolutionary equations with several fractional derivatives. *Mathematical Notes of NEFU*, 2021, vol. 28, no. 3, pp. 85–104 (in Russian). doi: 10.25587/SVFU.2021.75.46.006
10. Boyko K.V., Fedorov V.E. The Cauchy problem for a class of multi-term equations with Gerasimov – Caputo derivatives. *Lobachevskii J. Math.*, 2022, vol. 43, no. 6, pp. 1293–1302. doi: 10.1134/S1995080222090049
11. Boyko K.V., Fedorov V.E. Inverse problem for a class of degenerate evolutionary equations with several Gerasimov–Caputo derivatives. *Itogi Nauki i Tekhniki. Ser. Sovremennaya Matematika i ee Prilozheniya. Tematicheskie Obzory*, 2022, vol. 213, pp. 38–66 (in Russian). doi: 10.36535/0233-6723-2022-213-38-46
12. Fedorov V.E., Turov M.M. The defect of a Cauchy type problem for linear equations with several Riemann–Liouville derivatives. *Sib. Math. J.*, 2021, vol. 62, no. 5, pp. 925–942. doi: 10.1134/S0037446621050141

13. Turov M.M. Quasilinear equations with several Riemann–Liouville derivatives of arbitrary orders. *Chelyabinskii Fiz.-Mat. Zhurn.*, 2022, vol. 7, no. 4, pp. 434–446 (in Russian). doi: 10.47475/2500-0101-2022-17404
14. Gerasimov A.N. Generalization of linear laws of deformation and its applications to problems of inner friction. *Prikladnaya Matematika i Mekhanika*, 1948, vol. 12, no. 3, pp. 251–260 (in Russian).
15. Caputo M. Linear model of dissipation whose Q is almost frequency independent II. *Geophysical J. International*, 1967, vol. 13, no. 5, pp. 529–539. doi: 10.1111/j.1365-246X.1967.tb02303.x
16. Novozhenova O.G. Life and science of Alexey Gerasimov, one of the pioneers of fractional calculus in Soviet Union. *Frac. Calcul. Appl. Anal.*, 2017, vol. 20, no. 3, pp. 790–809. doi: 10.1515/fca-2017-0040
17. Bajlekova E.G. *Fractional Evolution Equations in Banach Spaces*: PhD thesis. Eindhoven, Eindhoven University of Technology, 2001, 107 p.
18. Kato T. *Perturbation Theory for Linear Operators*. Berlin, Heidelberg, Springer-Verlag, 1966. doi: 10.1007/978-3-642-66282-9. Translated to Russian under the title *Teoriya vozmushcheniya lineinykh operatorov*, Moscow, Mir Publ., 1972, 740 p.
19. Plekhanova M.V., Baybulatova G.D. *Semilinear equations in Banach spaces with lower fractional derivatives*. In: Springer Proc. Math. Stat., 2019, pp. 81–93. doi: 10.1007/978-3-030-26987-6_6
20. Hassard B.D., Kazarinoff N.D., Wan Y.-H. *Theory and applications of hopf bifurcation*. Cambridge, Cambridge Univ. Press, 1981, 311 p. doi: 10.1002/zamm.19820621221. Translated to Russian under the title *Teoriya i prilozheniya bifurkatsii rozhdeniya tsikla*, Moscow, Mir Publ., 1985, 280 p.

Received February 28, 2023

Revised March 15, 2023

Accepted March 20, 2023

Funding Agency: This work was supported by the RF President’s Grant for State Support of Leading Scientific Schools (project no. 2708.2022.1.1).

Vladimir Evgenyevich Fedorov, Dr. Phys.-Math. Sci., Prof., Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: kar@csu.ru.

Kseniya Vladimirovna Boyko, doctoral student, Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: kvboyko@mail.ru.

Cite this article as: V. E. Fedorov, K. V. Boyko. Quasilinear equations with a sectorial set of operators at Gerasimov–Caputo derivatives. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 2, pp. 248–259.