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AN ADAPTIVE ALGORITHM FOR A STABLE ONLINE IDENTIFICATION OF A DISTURBANCE IN A FRACTIONAL-ORDER SYSTEM ON AN INFINITE TIME HORIZON

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The problem of online identification of an uncontrolled external disturbance (noise) in a system of differential equations with a fractional Caputo derivative is considered on an infinite time horizon. Information on the position of the system is available for observations only during its functioning, and only a part of the coordinates of the system's phase vector can be measured. The case of measuring all phase coordinates is also considered. The measurements are carried out at discrete, sufficiently frequent times with a certain error. Therefore, the problem of finding the unknown disturbance is ill-posed. To solve it, an adaptive online identification algorithm is constructed using the dynamic inversion approach, which is based on a combination of regularization methods and constructions of positional control theory. In particular, we use the Tikhonov regularization method with a smoothing functional of special form and the Krasovskii extremal aiming method. The algorithm is based on the choice of an appropriate auxiliary control system and a feedback control law in this system. The proposed algorithm approximates the external disturbance and is stable under information noises and computational errors. A model example demonstrating the application of the developed technique is considered.

Keywords: online identification, external disturbance, Caputo fractional derivative, infinite time interval.

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