Trudy Instituta Matematiki i Mekhaniki UrO RAN

Vol. 29 No. 2 2023

MSC: 34A30, 34C10, 34D05

DOI: 10.21538/0134-4889-2023-29-2-157-171

ON ESSENTIAL VALUES OF EXPONENTS OF OSCILLATION FOR SOLUTIONS OF A LINEAR HOMOGENEOUS TWO-DIMENSIONAL DIFFERENTIAL SYSTEM

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In this paper, we study various types of exponents of oscillations for solutions of linear homogeneous differential systems with continuous bounded coefficients. The calculation of the exponents of oscillation is carried out by averaging the number of zeros (or signs, or roots, or hyper roots) of the projection of a solution x of a differential system onto any straight line, and this line is chosen so that the resulting average value is minimal: if the minimization is performed before (after) the averaging, then weak (strong, respectively) exponents of oscillation are obtained. In the calculation of the exponents of oscillation for a solution y of a linear homogeneous n-th order differential equation, a transition to the vector function $x = (y, \dot{y}, \dots, y^{(n-1)})$ is carried out. In the first part of the paper, for any preassigned natural number N, a two-dimensional periodic linear differential system is constructed, which has the property that its spectra of all upper and lower strong and weak exponents of oscillation of strict and nonstrict signs, zeros, roots, and hyper roots contain the same set, consisting of N different essential values, both metrically and topologically. Moreover, all these values are implemented on the same set of solutions of the constructed system, that is, for each solution from this set, all the exponents of oscillations coincide with each other. In the second part of the paper, a similar theorem on the existence of a two-dimensional differential system with a countable set of essential (both metrically and topologically) values of exponents of oscillation is proved. In constructing the mentioned systems and proving the required results, we use analytical methods of the qualitative theory of differential equations and methods of the theory of perturbations of solutions of linear differential systems, in particular, the author's technique for controlling the fundamental matrix of solutions of such systems in one special case.

Keywords: differential equation, linear system, oscillation, number of zeros, exponents of oscillation, Sergeev's frequency.

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Received February 2, 2023
Revised April 17, 2023
Accepted April 24, 2023

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Cite this article as: A. Kh. Stash. On essential values of exponents of oscillation for solutions of a linear homogeneous two-dimensional differential system. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 2, pp. 157–171.