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ONE APPROACH TO THE SOLUTION OF THE WAVE EQUATION FOR DIELECTRIC NONMAGNETIC MEDIA

L. I. Rubina

For dielectric nonmagnetic media, we consider a wave equation obtained from the system of Maxwell's equations in the framework of nonlinear optics with the classical approach. The equation describes the dynamics of the radiation field in glasses, liquids, gases, and many crystals. The dynamics of the electric field of radiation can be analyzed only with the knowledge of the polarization response of the medium to the force action of this field. Therefore, the wave equation can be considered underdetermined. It contains terms depending on the intensity $E = E(x, y, z, t)$ of the electric field of the radiation and a term depending on the polarization response of the medium $P = P(x, y, z, t)$. We propose a method for solving this underdetermined equation. Since the polarization response of the medium is caused by the force action of the electric field of radiation, we assume that the equation $P = P(E)$, $E(x, y, z, t) = \text{const}$, defines a level surface of the function P. Under this assumption, the wave equation reduces to an ODE. The independent variable in the ODE is the function E. The function $E = E(x, y, z, t)$ is found by solving the first-order partial differential equation (the basic equation) $E_t = f_0(E)$. The solution of the ODE and the form of $E = E(x, y, z, t)$ (hence, the dynamics of the radiation field) depend on the choice of an arbitrary function $f_0(E)$. The form of $E = E(x, y, z, t)$ and $P = P(E)$ is written for four functions $f_0(E)$. These solutions are found up to an arbitrary constant.

Keywords: wave equation, polarization, ODE system, functional arbitrariness.

REFERENCES

1. Bahaa E.A. Saleh, Teich M.C. *Fundamentals of Photonics*, 3 rd. ed. Wiley, 2019, 1520 p. ISBN: 9781119506874.
2. Sazonov S.V. On optical solitones of different durations. *Bulletin of Kazan State Univ., Phys. Math. Sci.,* 2008, vol. 150, no. 2, pp. 29–37 (in Russian).
3. Sazonov S.V. On nonlinear optics of ultimately short impulses. *Optics and Spectroscopy*, 2022, vol. 130, no. 12, pp. 1846–1855 (in Russian). doi: 10.21883/OS.2022.12.54090.45-22
4. Agranat M.B., Anisimov S.I., Makshantsev B.I. The anomalous thermal radiation from metals produced by ultrashort laser pulses. I, *Appl. Phys. B*, 1988, vol. 47, no. 3, pp. 209–221. doi: 10.1007/BF00697339
5. Akhmanov S.A., Vorontsov M.A., Ivanov V.Yu. Large-scale transverse nonlinear interactions in laser beams, new types of nonlinear waves, onset of "optical turbulence". *Letters to JETP,* 1988, vol. 47, no. 12, pp. 707–711.
6. Akaev A.A., Mayorov S.A. *Opticheskie metody obrabotki informatsii* [Optical methods of information processing], Moscow, Vysshaya Shkola Publ., 1988, 238 p.
7. Akimova I.G., Razgulin A.V. Rotational waves in an optical system with diffraction and rotation of spatial arguments. *Vestnik of Moscow State Univ., Ser. 15: Comput. Math. and Cybernetics,* 1999, no. 2, pp. 20–25 (in Russian).
8. Ryskin N. M., Trubetskov D. I. *Nelineinye volny* [Nonlinear waves]. Moscow, Fizmatlit Publ., 2000, 272 p.
9. Ovsiannikov L.V. *Group Analysis of Differential Equations.* NY, Acad. Press, 1982, 416 p. doi: 10.1016/C2013-0-07470-1 Original Russian text was published in Ovsiannikov L.V., Gruppovoi analiz differentsial'nykh uravnenii, Moscow, Nauka Publ., 1978, 399 pp.
10. Courant R., Hilbert D. *Methods of mathematical physics. Vol. 2: Partial differential equations,* NY, Interscience, 1962, 830 p. Translated to Russian under the title Metody matematicheskoi fiziki: Uravneniya v chastnykh proizvodnykh, Moscow, Mir Publ., 1964, 831 p.

11. Clarkson P.A., Kruskal M.D. New similarity reductions of the Boussinesq equation. *J. Math. Phys.,* 1989, vol. 30, no. 10, pp. 2201–2213. doi: 10.1063/1.528613
12. Sidorov A.F., Gavrilushkin I.B. On a class of solutions of a nonlinear equation for the velocity potential *J. Appl. Math. Mech.,* 1974, vol. 38, no. 2, pp. 237–244. doi: 10.1016/0021-8928(74)90063-X
13. Sidorov A.F. On some representations of solutions of quasilinear hyperbolic equations. In: *Computational Methods of Continuum Mechanics*, 1975, vol. 6, no. 4, pp. 106–115 (in Russian).
14. Sidorov A.F. On some classes of solutions to equations of unsteady filtration. In: *Computational Methods of Continuum Mechanics*, 1984, vol. 15, no. 2, pp. 121–133 (in Russian).
15. Sidorov A.F. Analytic representations of solutions of nonlinear parabolic equations of time-dependent filtration *Dokl. Academy of Sciences of the USSR,* 1985, vol. 31, pp. 40–44.
16. Rubina L.I.,Ulyanov O.N. Solving nonlinear partial differential equations by the geometric method. *Trudy Inst. Math. Mekh. UrO RAN*, 2012, vol. 18, no. 2, pp. 265–280 (in Russian).
17. Sidorov A.F., Shapeev V.P., Yanenko N.N. *Metod differentsial'nykh svyazei i ego prilozheniya k gazovoi dinamike* [The method of differential constraints and its applications to gas dynamics], Novosibirsk, Nauka Publ., 1984, 272 p.
18. Bruce M., Hua X. An approximate link equation for the direct-detected optical PPM link. *IPN Progress Report,* 2014, No. 42-199A. 14 p.
19. Boroson D.M., Robinson B.S., Murphy D.V., Burianek D.A., Khatri F., Kovalik J. M., Sodnik Z., Cornwell D. M. Overview and results of the lunar laser communication demonstration. In: *Proc. of the SPIE*, 2014, vol. 8971, article no. 8971OS, pp. 8971OS–8971OS-11. doi: 10.1117/12.2045508
20. Losev D.V., Bardashov D.S., Bykov A.G. Excitation of a semiconductor diode by a short pulse *Izvestiya Vuzov. Physics*, 2015, vol. 58, no. 8-2, pp. 147–150 (in Russian).
21. Alimenkov I.V., Pchelkina Yu.Zh. Integration in elementary funktions of two-way pulse–propagation equation in optical fibers for power nonlinearity. *Computer Optics*, 2014, vol. 38, no. 3, pp. 377–379 (in Russian).
22. Rubina L I., Ulyanov O.N. On the question of differences in the behavior of solutions of linear and nonlinear heat equations *Bulletin of South Ural State Univ. Series Maths. Mechanics. Physics,* 2013, vol. 5, no. 2, pp. 52–59 (in Russian).
23. Rubina L. I., Ulyanov O. N. On some method for solving a nonlinear heat equation. *Sib. Math. J.*, 2012, vol. 53, no. 5, pp. 872–881. doi: 10.1134/S0037446612050126
24. Rubina L. I., Ulyanov O. N. On one approach to solving inhomogeneous partial differential equations *Bulletin of the Udmurt University. Math. Mechanics. Computer science,* 2017, vol. 27, no. 3, pp. 355–364 (in Russian). doi: 10.20537vm170306

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Ljudmila Il'inichna Rubina, Cand. Sci (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: rli@imm.uran.ru .

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