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**SIMPLE INVARIANT SOLUTIONS OF THE DYNAMIC EQUATION
FOR A MONATOMIC GAS****R. F. Nikonorova**

We consider a system of gas dynamics equations with the state equation of a monatomic gas. The equations admit a group of transformations with a 14-dimensional Lie algebra. We consider 4-dimensional subalgebras containing the projective operator from the optimal system of subalgebras. The invariants of the basis operators are computed. Eight simple invariant solutions of rank 0 are obtained. Of these, four physical solutions specify a gas motion with a linear velocity field and one physical solution specifies a motion with a linear dependence of components of the velocity vector on two space coordinates. All these solutions except one have variable entropy. The motion of gas particles as a whole is constructed for the isentropic solution. The solutions obtained have a density singularity on a constant or moving plane, which is a boundary with vacuum or a wall.

Keywords: gas dynamics equations, projective operator, invariant solution.

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