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ON HILBERT SPACES OF SEQUENCES FORMED BY VALUES OF FUNCTIONS FROM THE BARGMANN–FOCK SPACE

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We study Hilbert spaces of sequences formed by values of functions from the Bargmann–Fock space F, which consists of entire functions whose square modulus is summable on the plane $\mathbb{C}$ with measure $d\sigma(z) := (1/\pi)e^{-|z|^2}\,dv(z)$, where $dv(z)$ is an area element:

$$\|f\|_F^2 = \int_{\mathbb{C}} |f(z)|^2\,d\sigma(z) < \infty \quad \forall f \in F.$$

The space $\overline{F}$ consists of complex conjugates of functions from F, and $\|\overline{f}\|_{\overline{F}} = \|f\|_F\ \forall f \in F$. We consider classes of countable sets $\Omega_0 \subset \mathbb{C}$ of the form

$$\Omega_0 \stackrel{\text{def}}{=} \{z \in \mathbb{C}\colon\ z = a\cdot n + ib\cdot m,\ a\cdot b = \pi\ \forall n, m \in \mathbb{Z}\},$$

where a and b are some fixed (depending only on the set Ω_0) nonzero real numbers. The sets Ω_0 are called von Neumann lattices. For a real number $k > 1$, we form the set $\Omega_0^k \stackrel{\text{def}}{=} k\cdot\Omega_0$. We establish that the space of sequences of complex numbers V_k formed by the traces of functions from some subspace F^k of the space F on the set Ω_0^k is equivalent to the space of sequences of complex numbers U_k formed by the traces of functions from a subspace $\overline{F}^k$ of the space $\overline{F}$ on the set Ω_0^k. The spaces $\overline{F}^k$ and $\overline{F}$ consist of complex conjugates of the functions from the spaces F^k and F, respectively. Moreover, the norms in the spaces V_k and U_k are induced by the norms of the spaces F^k and $\overline{F}^k$. To derive the main results of the paper, we use the result of K. Seip on discrete sampling sets of the Bargmann–Fock space. The results of the authors related to the questions of the coincidence or equivalence of Hilbert spaces with a reproducing kernel are applied. Here the notion of consistency of two complete systems of functions, introduced earlier by the authors, plays an important role. The paper presents counterexamples. We construct nonequivalent Hilbert spaces of complex numbers V and U that are the traces on some discrete subset of the complex plane of functions from F that are not equivalent.

Keywords: decomposition systems similar to orthogonal ones, Hilbert space with reproducing kernel, problem of describing a dual space, Bargmann–Fock space, exponential frame, von Neumann lattice.

REFERENCES

1. Aronszajn N. Theory of reproducing kernels. *Trans. Amer. Math. Soc.*, 1950, vol. 68, no. 3, pp. 337–404. doi: 10.1090/S0002-9947-1950-0051437-7
2. Bargmann V. On a Hilbert space of analytic functions and an associated integral transform. Part I. *Comm. Pure Appl. Math.*, 1961, vol. 14, no. 3, pp. 187–214. doi: 10.1002/cpa.3160140303
3. Seip K., Wallsten R. Density theorems for sampling and interpolation in the Bargmann–Fock space II. *J. reine angew. Math.*, 1992, vol. 429, pp. 107–113. doi: 10.1515/crll.1992.429.107
4. Seip K. Density theorems for sampling and interpolation in the Bargmann-Fock space I. *J. reine angew. Math.*, 1992, vol. 429, pp. 91–106. doi: 10.1515/crll.1992.429.91
5. Lukashenko T.P. Properties of expansion systems similar to orthogonal ones. *Izvestiya: Mathematics*, 1998, vol. 62, no. 5, pp. 1035 –1054. doi: 10.1070/IM1998v062n05ABEH000215
6. Riesz F., Szőkefalvi-Nagy B. *Leçons d'analyse fonctionnelle,* Paris, Gauthier-Villars, 1975, 488 p. ISBN 2040018700. Translated to Russian under the title *Lektsii po funktsional'nomu analizu*, Moscow, Mir Publ., 1979, 588 p.
7. Napalkov V.V. (Jr.) On an orthosimilar system in the space of analytical function and a problem of describing the dual space. *Ufa Math. J.*, 2011, vol. 3, no. 1, pp. 30–41.

8. Dunford N.J., Schwartz J.T. *Linear Operators. Part I: General Theory*, John Wiley & Sons Inc, 1958. ISBN: 9780470226056. Translated to Russian under the title *Lineinye operatory. Obshchaya teoriya*, Moscow, Inostr. Liter Publ., 1962, 896 p.
9. Napalkov V.V. (Jr.) Orthosimilar expansion systems in space with reproducing kernel. *Ufa Math. J.*, 2013, vol. 5, no. 4, pp. 88-100. doi: 10.13108/2013-5-4-88
10. Napalkov V.V., Napalkov V.V., Jr. On the coincidence of reproducing kernel Hilbert spaces connected by a special transformation, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 2, pp. 149–159 . doi: 10.21538/0134-4889-2019-25-2-149-159
11. Kantorovich L.V., Akilov G.P., *Functional Analysis*, Pergamon Press, 1982, 604 p. ISBN 0080264867. Original Russian text published in Kantorovich L.V., Akilov G.P. *Funktsional'nyi analiz*, Moscow, Nauka Publ., 1977, 741 p.
12. Daubechies I., Grossmann A. Frames in the Bargmann space of entire functions. *Comm. Pure Appl. Math.*, 1988, vol. 41, no. 2, pp. 151–164. doi: 10.1002/cpa.3160410203
13. Lyubarskii Y.I. Frames in the Bargmann space of entire functions In: *Entire and Subharmonic Functions*, ed. Boris Ya Levin, vol. 11 of *Adv. Sov. Math.*, Providence, Amer. Math. Soc., 1992, pp. 167–180. doi: 10.1090/advsov/011
14. Napalkov V. V.; Napalkov V. V. Jr. On isomorphism of reproducing kernel Hilbert spaces. *Dokl. Math.*, 2017, vol. 95, no. 3, pp. 270–272. doi: 10.1134/S1064562417030243
15. Newman D.J., Shapiro H.S. Certain Hilbert spaces of entire functions. *Bull. Amer. Math. Soc.*, 1966, vol. 72, no. 6, pp. 971-977. doi: 10.1090/S0002-9904-1966-11608-7

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