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OPTIMAL RECOVERY ON CLASSES OF FUNCTIONS ANALYTIC IN A ANNULUS

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Let $C_{r,R}$ be a annulus with boundary circles γ_r and γ_R centered at zero; its inner and outer radii are r and R, respectively. On the class of functions analytic in the annulus $C_{r,R}$ with finite L^2-norms of the angular limits on the circle γ_r and of the nth derivatives (of the functions themselves for $n = 0$) on the circle γ_R, we study interconnected extremal problems for the operator ψ_ρ^m that takes the boundary values of a function on γ_r to its restriction (for $m = 0$) or the restriction of its mth derivative (for $m > 0$) to an intermediate circle γ_ρ, $r < \rho < R$. The problem of the best approximation of ψ_ρ^m by linear bounded operators from $L^2(\gamma_r)$ to $C(\gamma_\rho)$ is solved. A method for the optimal recovery of the mth derivative on a intermediate circle γ_ρ from L^2-approximately given values of the function on the boundary circle γ_r is proposed and its error is found. The Hadamard–Kolmogorov exact inequality, which estimates the uniform norm of the mth derivative on an intermediate circle γ_ρ in terms of the L^2-norms of the limit boundary values of the function and the nth derivative on the circles γ_r and γ_R, is derived.

Keywords: analytic functions, Hadamard three-circle theorem, Kolmogorov's inequality, optimal recovery.

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