ASYMPTOTICS OF A SOLUTION TO AN OPTIMAL CONTROL PROBLEM WITH INTEGRAL CONVEX PERFORMANCE INDEX, CHEAP CONTROL, AND INITIAL DATA PERTURBATIONS

A. R. Danilin, A. A. Shaburov

A. R. Danilin, A. A. Shaburov. Asymptotics of a solution to an optimal control problem with integral convex performance index, cheap control, and initial data perturbations.

We consider an optimal control problem in the class of piecewise continuous controls with smooth geometric constraints for a linear system with constant coefficients and an integral convex performance criterion containing two small parameters (the first of them multiplying the integral term, and the second in the initial data). Such problems are called cheap control problems. It is shown that a problem with a terminal performance index will be the limit one. It is established that if the limit problem is actually one-dimensional whereas the initial problem is not, then the asymptotics of the solution can be more complicated. In particular, the asymptotics of the solution may have no expansion in the Poincare sense in any asymptotic sequence of rational functions of the small parameter or its logarithms.

Keywords: optimal control, cheap control, asymptotic expansion, small parameter.

REFERENCES

1. Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishchenko E.F. *The Mathematical Theory of Optimal Processes.* NY: London: Sydney: John Wiley and Sons, Inc., 1962. 360 p. doi:10.1002/zamm.19630431023 Original Russian text was published in Matematicheskaya teoriya optimal'nykh protsessov, Moscow, Phys. Math. Liter. Publ., 1961, 391 p.

2. Krasovskii N.N. *Teoriya upravleniya dvizheniem: Lineinye sistemy* [The Theory of the Control of Motion: Linear Systems]. Moscow, Nauka Publ., 1968, 476 pp.

3. Lee E.B., Markus L. *Foundations of Optimal Control Theory.* NY; London–Sydney: John Wiley and Sons, Inc., 1967, 588 p. Translated to Russian under the title Osnovy teorii optimal'nogo upravleniya, Moscow, Nauka publ., 1972, 576 pp.

4. Dmitriev M.G., Kurina G.A. Singular perturbations in control problems. *Automation and Remote Control,* 2006, vol. 67, no. 1, pp. 1–43. doi:10.1134/S0005117906010012

5. Zhang Y., Naidu D.S., Cai C., Zou Y. Singular perturbation and time scales in control theories and applications: an overview 2002–2012. *Internat. J. of Information and Systems Sciences*, 2014, vol. 9, no. 1, pp. 1–36.

6. Kurina G.A., Kalashnikova M.A. Singularly perturbed problems with quick variables of different paces. *Avtomatika i Telemekhanika,* 2022, no. 11, pp. 3–61 (in Russian). doi:10.31857/S0005231022110010

7. Glizer V.Ya., Dmitriev M.G. Asymptotics for the solution of a singularly perturbed Cauchy problem arising in the theory of optimal control. *Differ. Uravn.,* 1978, vol. 14, no. 4, pp. 601–612 (in Russian).

8. Hoai N.T. Asymptotic solution of a singularly perturbed linear–quadratic problem in critical case with cheap control. *J. Optim Theory Appl.*, 2017, vol. 175, no. 2, pp. 324–340. doi: 10.1007/s10957-017-1156-6

9. Kalashnikova M.A., Kurina G.A. Direct scheme for the asymptotic solution of linear-quadratic problems with cheap controls of different costs. *Differ. Equat.,* 2019, vol. 55, no. 1, pp. 84–104. doi: 10.1134/S0012266119010099

10. Danilin A.R., Il'in A.M. The asymptotical behavior of the solution to the time-optimal problem for a linear system under perturbation of initial data. *Dokl. Math.,* 1996, vol. 54, no. 2, pp. 673–675.

11. Danilin A.R., Il'in A.M. On the structure of the solution of a perturbed time-optimal control problem. *Fundam. Prikl. Mat.*, vol. 4, no. 3, pp. 905–926 (in Russian).

12. Danilin A.R., Shaburov A.A. Asymptotic expansion of an optimal control problem with convex integral performance index and cheap control. *Sib. J. Industr. Math.*, 2022, vol. 25, no. 3, pp. 5–13 (in Russian). doi:10.33048/SIBJIM.2021.25.301

13. Rockafellar R.T. *Convex Analysis,* Princeton, Princeton Univ. Press., 1970, 451 p. Translated to Russian under the title Vypuklyi analiz, Moscow, Mir Publ., 1973, 471 p.

14. Galeev E.M., Tikhomirov V.M. *Kratkii kurs teorii ekstremal'nykh zadach* [Short course of extremal problems theory]. Moscow, Moscow State Univ. Publ., 1989. 204 p.

15. Danilin A.R., Kovrizhnykh O.O. Asymptotics of the solution to a singularly perturbed timeoptimal control problem with two small parameters. *Trudy Inst. Math. Mech. UrO RAN*, 2019, vol. 25, no. 2, pp. 88–101 (in Russian). doi: 10.21538/0134-4889-2019-25-2-88-101

16. Danilin A.R., Kovrizhnykh O.O. Asymptotics of a solution to a time-optimal control problem with an unbounded target set in the critical case. *Trudy Inst. Math. Mech. UrO RAN*, 2022, vol. 28, no. 1, pp. 58–73 (in Russian). doi: 10.21538/0134-4889-2022-28-1-58-73

Aleksei Rufimovich Danilin, Dr. Phys.-Math. Sci., Prof., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: dar@imm.uran.ru .

Aleksandr Aleksandrovich Shaburov, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: alexandershaburov@mail.ru .