OPERATOR ESTIMATES IN TWO-DIMENSIONAL PROBLEMS WITH A FREQUENT ALTERNATION IN THE CASE OF SMALL PARTS WITH THE DIRICHLET CONDITION

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A two-dimensional boundary value problem is studied for a scalar elliptic second-order equation of the general form with frequent alternation of boundary conditions. The alternation is defined on small, closely spaced parts of the boundary, on which the Dirichlet boundary condition and the nonlinear third boundary condition are set alternately. The distribution and size of these segments are arbitrary. The case is considered when, upon averaging, the Dirichlet boundary condition completely disappears and only the original nonlinear third boundary condition remains. The main result is estimates for the W_2^1- and L_2-norms of the difference between the solutions of the perturbed and averaged problems, which are uniform in the L_2-norm of the right-hand side and characterize the rate of convergence. It is shown that these estimates are exact in the order of smallness.

Keywords: two-dimensional boundary value problem, elliptic equation, frequent alternation, homogenization, operator estimate.

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