EXACT CONSTANTS IN JACKSON–STECHKIN INEQUALITY IN L^2 WITH A POWER-LAW WEIGHT

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In this paper, we have solved several extremal problems of the best mean-square approximation of function f, on the semiaxis with a power-law weight, which can be used to solve various problems. Sharp Jackson–Stechkin type inequalities are obtained on some classes of functions in which the values of the best approximations are estimated from above through moduli of smoothness of the k-th order.

Keywords: exact constants in Jackson–Stechkin inequality, moduli of smoothness, best approximations, Bessel function.

REFERENCES

1. Arestov V.V., Babenko A.G, Deikalova M.V., Horv'ath A. Nikol'skii Inequality Between the Uniform Norm and Integral Norm with Bessel Weight for Entire Functions of Exponential Type on the Half-Line. *Anal. Math.,* 2018, vol. 44, no. 1, pp. 21–42. doi:10.1007/s10476-018-0103-6

2. Arestov V.V., Popov V.Yu. Jackson inequalities on a sphere in L_2. *Russian Math. (Iz. VUZ),* 1995, vol. 39, no. 8, pp. 11–18.

3. Babenko A.G. Exact Jackson–Stechkin inequality in the space $L_2(\mathbb{R}^m)$. *Trudy Inst Math. Mekh.,* 1998, vol. 5, pp. 183–198 (in Russian).

4. Berdysheva E.E. Two related extremal problems for entire functions of several variables. *Math. Notes,* 1999, vol. 66, no. 3, pp. 271–282. doi:10.1007/BF02676434

5. Betancor J.J., Rodriguez-Mesa L. On the Besov–Hankel spaces. *J. Math Soc. Japan.,* 1998, vol. 50, no. 3, pp. 781–788. doi:10.2969/jmsj/05030781

6. Chernykh N.I. Jackson's inequality in L_2. *Proc. Steklov Inst. Math.,* 1967, vol. 88, pp. 75–78.

7. Chernykh N.I. The best approximation of periodic functions by trigonometric polynomials in L_2. *Math. Notes,* 1967, vol. 2, no. 5, pp. 803–808. doi:10.1007/BF01093942

8. Esmaganbetov M.G. Widths of classes from $L_2[0, 2\pi]$ and the minimization of exact constants in Jackson-type inequalities. *Math. Notes,* 1999, vol. 65, no. 6, pp. 689–693. doi:10.1007/BF02675582

9. Gorbachev D.V. Extremum problems for entire functions of exponential spherical type. *Math. Notes,* 2000, vol. 68, no. 2, pp. 159–166. doi:10.1007/BF02675341

10. Gorbachev D.V. An estimate of an optimal argument in the sharp multidimensional Jackson–Stechkin L_2-inequality. *Proc. Steklov Inst. Math.,* 2015, vol. 288, no. 1 (suppl.), pp. 70–78. doi:10.1134/S008154381502008X

11. Hirschman I.I. Variation diminishing Hankel transforms. *J. d'analyse Mathematique,* 1960, vol. 8, no. 1, pp. 307–336. doi:10.1007/BF02786855

12. Ibragimov I.I., Nasibov F.G. The estimation of the best approximation of a summable function on the real axis by means of entire functions of finite degree. *Soviet Math.,* 1970, vol. 11, p. 1332–1336.

13. Ivanov V.I. On the Sharpness of Jackson's Inequality in the Spaces L_p on the Half-Line with Power Weight. *Math. Notes,* 2015, vol. 98, no. 5, pp. 742–751. doi: 10.1134/S0001434615110048

14. Korneychuk N.P. *Exact constants in approximation theory.* In: Encyclopedia of Mathematics and its Applications. Cambridge, Cambridge Univ. Press, 2009, 468 p. ISBN: 9780521111560. Original Russian text was published in Korneychuk N.P., Moscow: Nauka Publ., 1987, 424 p.

15. Levitan B.M. Expansion in Fourier series and integrals in Bessel functions. *Uspekhi Mat. Nauk* (Russian Math. Surveys), 1951, vol. 6, no. 2(42), pp. 102–143 (in Russian).

16. Li J., Liu Y.P. The Jackson Inequality for the Best L_2-Approximation of Functions on [0,1] with the Weight x. *Numer. Math. Theory Methods Appl.,* 2008, vol 1, no. 3, pp. 340–356.

17. Moskovsky A.V. Jackson's theorems in the spaces $L_p(R_n)$ and $L_p, \lambda(R+)$. *Izvestiya of Tula State Univ. Ser. Math., Mekh., Inform.,* 1997, vol. 3, no. 1, pp. 44–70 (in Russian).

18. Moskovsky A.V. Jackson's theorems in the spaces $L_p(R_n)$, $L_p, \lambda(R+)$, $1 \le p \le 2$. *Izvestiya of Tula State Univ. Ser. Math., Mekh., Inform.,* 1998, vol. 4, no. 1, pp. 97–101 (in Russian).

19. Nikol'skii S.M., Lizorkin P.I. Approximation of functions on the sphere. *Math. USSR. Izv.,* 1988, vol. 30, no. 3, pp. 599–614. doi:10.1070/IM1988v030n03ABEH001032

20. Platonov S.S. Bessel harmonic analysis and approximation of functions on the half-line. *Izv. Math.,* 2007, vol. 71, no. 5, pp. 1001–1048. doi:10.1070/IM2007v071n05ABEH002379

21. Popov V.Yu. On the best mean square approximations by entire functions of exponential type. *Soviet Math. (Iz. VUZ),* 1972, vol. 6, pp. 65–73 (in Russian).

22. Shabozov M.Sh., Tukhliev K., Murodov K.N. Exact estimates for the rate of convergence of Fourier–Bessel ranks and values of n-widths of some classes of functions. *Questions of comput. and appl. math.,* 2015, vol. 2, pp. 39–47 (in Russian).

23. Rustamov Kh.P. On approximation of functions on the sphere. *Izv. Math.,* 1994, vol. 43, no. 2, pp. 311–329. doi:10.1070/IM1994v043n02ABEH001566

24. Shabozov M.Sh. Widths of Classes of Periodic Differentiable Functions in the Space $L_2[0, 2\pi]$. *Math. Notes,* 2010, vol. 87, no. 3–4, pp. 575–581. doi:10.1134/S0001434610030351

25. Shalaev V.V. Widths in L_2 of classes of differentiable functions defined by higher-order moduli of continuity. *Ukr. Math. J.,* 1991, vol. 43, no. 1, pp. 104–107. doi:10.1007/BF01066914

26. Shabozov M.Sh., Yusupov G.A. On the exact values of average ν-widths of some classes of entire functions. *Trudy Inst. Math. Mekh.,* 2012, vol. 18, no. 4, pp. 315–327 (in Russian).

27. Stein E.M., Weiss G. *Introduction to Fourier Analysis on Euclidean Spaces,* Princeton, Princeton Univ. Press, 1971, 312 pp. EISBN: 978-1-4008-8389-9 . Translated to Russian under the title Vvedenie v garmonicheskii analiz na evklidovykh prostranstvakh, Moscow, Mir publ., 1974, 328 pp.

28. Taikov L.V. Structural and constructive characteristics of functions in L_2. *Math. Notes,* 1979, vol. 25, no. 2, pp. 113–116. doi:10.1007/BF01142721

29. Trimeche K. Inversion of the Lions Transmutation Operators Using Generalized Wavelets. *Appl. Comput. Harmonic Anal.,* 1997, vol. 4, no. 1, pp. 97–112. doi:10.1006/acha.1996.0206

30. Trimeche K. *Generalized Harmonic Analysis and Wavelet Packets: An Elementary Treatment of Theory and Applications,* London, CRC Press, 2001, 320 pp. doi:10.1201/9781482283174

31. Vakarchuk S.B. Jackson-type inequalities for the special moduli of continuity on the entire real axis and the exact values of mean ν-widths for the classes of functions in the space $L^2(R)$. *Ukr. Math J.,* 2014, vol 66, no. 6, pp. 827–856. doi:10.1007/s11253-014-0977-9

32. Vakarchuk S.B., Shabozov M.Sh., Langarshoev M.R. On the best mean square approximations by entire functions of exponential type in $L_2(R)$ and mean ν-widths of some functional classes. *Russian Math. (Iz. VUZ),* 2014, vol. 58, no. 7, pp. 25–41. doi:10.3103/S1066369X14070032

33. Vakarchuk S.B., Shabozov M.Sh., Zabutnaya V.I. Structural characteristics of functions from L^2 and the exact values of widths of some functional classes. *J. Math. Sci.,* 2015, vol. 206, no. 1, pp. 97–114. doi:10.1007/s10958-015-2296-6

34. Watson G.N. *A treatise of the theory of Bessel functions,* Cambridge, Cambridge Univ. Press, 1945, vii+804 p. Translated to Russian under the title Teoriya besselevykh funktsii, Moscow, Inostr. Liter publ., 1949, 799 p.

35. Yudin V.A. Multidimensional Jackson Theorem in L_2. *Math. Notes,* 1981, vol. 29, no. 2, 158–162. doi:10.1007/BF01140931

36. Yudin V.A. Extremality properties of functions and designs on the torus. *Math. Notes,* 1997, vol. 61, no. 4, pp. 530–533. doi:10.1007/BF02355000

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