EXTREMAL INTERPOLATION IN THE MEAN
WITH OVERLAPPING AVERAGING INTERVALS
AND THE SMALLEST NORM OF A LINEAR DIFFERENTIAL OPERATOR

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The Yanenko–Stechkin–Subbotin problem of extremal functional interpolation in the mean is considered for sequences infinite in both directions on a uniform grid of the numerical axis with the smallest norm in the space $L_p(R)$ $(1 < p < \infty)$ of a linear differential operator $\mathcal{L}_n$ with constant coefficients. It is assumed that the generalized finite differences of each sequence corresponding to the operator $\mathcal{L}_n$ are bounded in the space l_p, the grid step h and the averaging step h_1 are related by the inequality $h < h_1 < 2h$, and the operator $\mathcal{L}_n$ is formally self-adjoint. Under these assumptions, in the case of odd n, the smallest norm of the operator is found exactly, and the extremal function is a generalized $\mathcal{L}$-spline whose knots coincide with the interpolation nodes. This work continues the research of this problem by Yu. N. Subbotin and the author started by Subbotin in 1965.

Keywords: extremal interpolation, splines, uniform grid, formally self-adjoint differential operator, minimum norm, splines.

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