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DIFFERENTIALLY INVARIANT SUBMODELS OF GAS DYNAMICS FOR THE FOUR-DIMENSIONAL SUBALGEBRA OF TRANSLATIONS

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Continuum models admit a Lie algebra of the group containing translations, Galilean transformations, rotations, and dilatation. Submodels are constructed for subalgebras of different dimensions. For dimensions 1, 2, and 3, these are invariant submodels. For subalgebras of dimension 4, invariant solutions given by finite formulas, partially invariant submodels, and also differentially invariant submodels are possible. For equations of gas-dynamic type, using the example of a four-dimensional subalgebra of translations, a method is proposed for constructing differentially invariant submodels of minimal rank. For this, the basis of differential invariants and operators of invariant differentiation are calculated. Independent differential invariants are chosen by virtue of the model equations, and the simplest representation of a nontrivial solution is determined. Substitution of the representation into the model equations gives an overdetermined system. Reduction to involution occurs by finding integrable combinations and alternative assumptions. As a result, exact solutions and submodels with ordinary differential equations are obtained for spatial, plane, and one-dimensional motions with a linear velocity field.

Keywords: gas dynamics, differentially invariant solutions, linear velocity field, reduction to involution.

REFERENCES

1. Ovsyannikov L.V. Lektsii po osnovam gazovoy dinamiki [Lectures on the fundamentels of gas dynamics]. Moskva, Izhevsk: Institut Komp'yuternykh Issledovaniy Publ., 2003. 336 p.
2. Ovsiannikov L.V. Group analysis of differential equations. NY: Acad. Press, 1982. 416 p. doi: 10.1016/C2013-0-07470-1 . Original Russian text published in L. V. Ovsyannikov Gruppovoi analiz differentsial'nykh uravnenii. Moscow: Nauka Publ., 1978.
3. Khabirov S.V. Classification of differential invariant submodels. *Siberian Math. J.*, 2004, vol. 45, pp. 562–579. doi: 10.1023/B:SIMJ.0000028621.02366.bf
4. Ovsyannikov L.V. Regular partially invariant submodels of the equations of gas dynamics. *J. Appl. Math. Mech.*, 1996, vol. 60, no. 6, pp. 969–978. doi: 10.1016/S0021-8928(96)00119-0

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