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ON INFINITE-HORIZON OPTIMAL EXPLOITATION OF A RENEWABLE RESOURCE

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We consider models of homogeneous and structured (for example, by age, gender, or other attribute) populations given by difference equations. The dynamics of a structured population in the absence of exploitation is given by the system of equations $x(k+1) = F\big(x(k)\big)$, $k = 0, 1, 2, \dots$; here $F(x)$ is a column vector with coordinates $f_1(x), \dots, f_n(x)$, which are real nonnegative continuous functions, and $x(k) = \big(x_1(k), \dots, x_n(k)\big)$, where $x_i(k)$, $i = 1, \dots, n$, is the amount of resource of the ith type or age class at time $k = 0, 1, 2, \dots$. A homogeneous population is given by the difference equation $x(k+1) = f\big(x(k)\big)$, $k = 0, 1, 2, \dots$. It is assumed that the population is subject to harvesting $u(k) = \big(u_1(k), \dots, u_n(k)\big) \in [0, 1]^n$ at fixed times $k = 0, 1, 2, \dots$, and this process can be controlled to achieve a certain result of resource harvesting. Thus, we consider the models of the exploited populations given by the systems of equations $x(k+1) = F\big((1 - u(k))x(k)\big)$, $k = 0, 1, 2, \dots$. We study the infinite-horizon problem of optimal harvesting of a renewable resource for stationary and general exploitation modes. The characteristics of resource harvesting are considered, the first of which is the harvesting efficiency equal to the limit as $k \to \infty$ of the ratio of the cost of the resource gathered in k harvestings to the amount of applied control (harvesting efforts). Another characteristic is the mean time profit, which is the limit as $k \to \infty$ of the arithmetic mean of the cost of the resource over k harvestings. We find the highest values of these characteristics and describe the harvesting strategies under which these values are attained. It is shown that if all possible controls are taken into account in population exploitation, then a value of harvesting efficiency greater than the highest efficiency on the set of stationary controls can be attained. On the other hand, the largest value of the mean time profit calculated on the set of all controls coincides with the largest value on the set of stationary controls and does not depend on $x(0)$. The results are illustrated by the examples of an exploited population given by a discrete logistic equation and a structured population consisting of two species.

Keywords: model of a population subject to harvesting, population exploitation modes, optimal exploitation, resource harvesting efficiency, average time profit.

REFERENCES

1. Scott Gordon H. The Economic Theory of a Common-Property Resource: The Fishery. *J. Polit. Econ.*, 1954, vol. 62, pp. 124–142. doi: 10.1016/s0092-8240(05)80048-5
2. Brauer F., Sanchez D.A. Constant rate population harvesting: equilibrium and stability. *Theor. Popul. Biol.*, 1975, vol. 8, no. 1, pp. 12–30. doi: 10.1016/0040-5809(75)90036-2
3. Freedman H.I., So J.W.-H. Persistence in discrete models of a population which may be subjected to harvesting // *Natural Resource Modeling,* 1987, vol. 2, no. 1, pp. 135–145. doi: 10.1111/j.1939-7445.1987.tb00029.x
4. Brites N.M., Braumann C.A. Fisheries management in random environments: Comparison of harvesting policies for the logistic model. *Fisheries Research,* 2017, vol. 195, pp. 238–246. doi: 10.1016/j.fishres.2017.07.016
5. Davydov A.A., Platov A.S. Optimal stationary solution for a model of exploitation of a population under intraspecific competition. *J. Math. Sci.,* 2014, vol. 201, no 6, pp. 746–751. doi: 10.1007/s10958-014-2023-8
6. Zelikin M.I., Lokutsievskiy L.V., Skopincev S.V. On optimal harvesting of a resource on a circle. *Math. Notes*, 2017, vol. 102, no. 3–4, pp. 521–532. doi: 10.1134/S0001434617090243
7. Neverova G.P., Abakumov A.I., Frisman E.Ya. Dynamic modes of exploited limited population: results of modeling and numerical study. *Mat. Biologija i Bioinformatika*, 2016, vol. 11, no. 1, pp. 1–13 (in Russian). doi: 10.17537/2016.11.1

8. Revutskaya O.L., Frisman E.Ya. Influence of stationary harvesting on development of a two-age population scenario. *Informatika i Sistemy Upravleniya*, 2017, vol. 53, no. 3, pp. 36–48 (in Russian). doi: 10.22250/isu.2017.53.36-48
9. Upmann T., Uecker H., Hammann L., Blasius B. Optimal stock–enhancement of a spatially distributed renewable resource. *J. Econ. Dynamics and Control,* 2021, vol. 123, article no. 104060. doi: 10.1016/j.jedc.2020.104060
10. Egorova A.V., Rodina L.I. On optimal harvesting of renewable resource from the structured population. *Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki*, 2019, vol. 29, no. 4, pp. 501–517 (in Russian). doi: 10.20537/vm190403
11. Belyakov A.O., Davydov A.A., Efficiency optimization for the cyclic use of a renewable resource. *Proc. Steklov Inst. Math. (Suppl.)*, 2017, vol. 299, suppl. 1, pp. 14–21. doi: 10.1134/S0081543817090036
12. Svirezhev Yu.M., Logofet D.O. *Stability of biological communities.* Moscow, Mir Publ., 1983, 319 p. ISBN 10: 0828523711. Original Russian text was published in Ustoichivost' biologicheskikh soobshchestv, Moscow, Nauka Publ., 1978, 352 p.
13. Rodina L.I. *Raznostnye uravneniya kak modeli biologicheskikh protsessov* [Difference equations as models of biological processes]. Vladimir, Vladimir. State Univ. Publ., 2022. 82 p.
14. Sharkovsky A.N., Kolyada S.F., Sivak A.G., Fedorenko V.V. *Dynamics of one-dimensional maps*, Dordrecht,, Springer, 1997, 262 p. doi: 10.1007/978-94-015-8897-3 . Original Russian text was published in Dinamika odnomernykh otobrazhenii, Kiev, Naukova Dumka Publ., 1989, 216 p.
15. Woldeab M.S., Rodina L.I. About the methods of renewable resourse extraction from the structured population, *Vestnik Rossiiskikh Universitetov. Matematika*, 2022, vol. 27, no. 137, pp. 16–26 (in Russian). doi: 10.20310/2686-9667-2022-27-137-16-26

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