ON AN ELEMENT-BY-ELEMENT DESCRIPTION OF THE MONOID OF ALL ENDOMORPHISMS OF AN ARBITRARY GROUPOID AND ONE CLASSIFICATION OF ENDOMORPHISMS OF A GROUPOID

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The problem of element-by-element description of the monoid of all endomorphisms of an arbitrary groupoid is considered. It is established that this monoid is decomposed into a union of pairwise disjoint classes of endomorphisms; these classes are called basic sets of endomorphisms. These sets of endomorphisms of a groupoid G are parameterized by mappings $\gamma : G \rightarrow \{1, 2\}$, which in this paper are called bipolar types (hereinafter, simply types). If some endomorphism lies in a basic set of type γ, then we say that it has type γ. Thus, we obtain a classification of all endomorphisms of a fixed groupoid (a bipolar classification of endomorphisms). A connection between the types of endomorphisms of two isomorphic groupoids is revealed. The basic set of endomorphisms need not be closed under composition. Groupoids are constructed in which some basic sets are closed. For each basic set, an endomorphism semigroup contained in this basic set is constructed. These semigroups in some cases degenerate into empty sets. Examples of groupoids are given in which the constructed endomorphism semigroups are nonempty. The constructed semigroups can be used to study the problem of element-by-element description of the monoid of all endomorphisms and to study the structure of the monoid of all endomorphisms.

Keywords: groupoid endomorphism, groupoid automorphism, groupoid, basic set of endomorphisms, bipolar classification of groupoid endomorphisms, monotypic endomorphism semigroups.

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