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**ZERO-ORDER ASYMPTOTICS FOR THE SOLUTION OF ONE TYPE  
OF SINGULARLY PERTURBED LINEAR–QUADRATIC CONTROL PROBLEMS  
IN THE CRITICAL CASE**

**G. A. Kurina, N. T. Hoai**

**G. A. Kurina, N. T. Hoai. Zero-order asymptotics for the solution of one type of singularly perturbed linear–quadratic control problems in the critical case.**

We consider a linear–quadratic control problem in which there is the second power of a small parameter at the derivative of the state variable and the first power of the parameter both in the control term of the state equation and at the quadratic form of the control variable in the performance index; moreover, the state equation represents a critical case of singular perturbation theory. A zero-order asymptotic expansion of the solution is constructed using the so-called direct scheme method, in which a postulated asymptotic expansion of the solution is substituted directly into the problem statement and problems for finding the asymptotic terms are stated.

Keywords: linear–quadratic control problem, singular perturbations, critical case, asymptotics of solution.

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