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ON A CONTROL SYNTHESIS IN AN ENHANCED EVASION PROBLEM FOR LINEAR DISCRETE-TIME SYSTEMS

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The evasion problem is considered for linear discrete-time systems with two controls that may have different aims. The aim of one of them is to exclude, regardless of the other, the trajectory hitting a given terminal set at a given final instant, and moreover to exclude the trajectory hitting a sequence of sets given at previous instants. We call it an enhanced evasion problem. Its special case is the problem of trajectory evasion from the terminal set at all instants. A method of control synthesis based on the construction of solvability tubes is presented. However, it is usually quite difficult to accurately construct them. Then it is assumed that the terminal and intermediate sets are parallelepipeds and both controls are bounded by parallelotope-valued constraints. A fast method of control synthesis based on the construction of a pair of polyhedral tubes with parallelepiped-valued cross-sections is proposed and justified. The proposed procedures are applicable for cases with possible degeneration or emptiness of the cross-sections at some instants. The cross-sections of the tubes and the control values are found from explicit formulas. Several variants of the control synthesis formula are given. Examples are given to illustrate the presented method.

Keywords: systems with uncertainties, evasion problem, control synthesis, polyhedral methods, parallelepipeds.

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