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**GENERALIZED ABSOLUTE CONVERGENCE OF FOURIER SERIES  
WITH RESPECT TO MULTIPLICATIVE SYSTEMS OF FUNCTIONS  
OF GENERALIZED BOUNDED FLUCTUATION****S. S. Volosivets, A. N. Mingachev**

The series of one-dimensional and two-dimensional Fourier coefficients with respect to multiplicative systems  $\chi$  (with a bounded generating sequence  $\mathbf{P} = \{p_i\}_{i=1}^{\infty}$ ) with weights satisfying Gogoladze–Meskhia type conditions are studied. Sufficient conditions for the convergence of such series are established for functions from different classes of generalized bounded fluctuation.

Keywords: absolute convergence, multiplicative system, double series, generalized bounded fluctuation.

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