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SHAPE PRESERVING CONDITIONS FOR INTEGRO QUADRATIC SPLINE INTERPOLATION IN THE MEAN

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Earlier, Yu. N. Subbotin considered the problem of interpolation in the mean, where the interpolated values of the function are replaced by averaged values on an interval. In his paper, the grid was uniform, but the space grid step could differ from the size of the averaging intervals. Subbotin investigated the existence of such splines and their convergence in different metrics. In the literature, splines of this type are also called integro or histosplines. The present paper considers such an interpolating in the mean quadratic spline on an arbitrary nonuniform grid of a closed interval, where the averaging intervals are the grid intervals. Sufficient conditions are obtained for the inheritance by an integro spline of certain properties of the approximated function such as nonnegativity, monotonicity, and convexity.

Keywords: integro spline, interpolation in the mean, shape preserving, quadratic splines.

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