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AN ALGORITHM FOR TAKING A BIPARTITE GRAPH TO THE BIPARTITE THRESHOLD FORM

V. A. Baranskii, T. A. Sen'chonok

A triple of different vertices (x, v, y) of a graph $G = (V, E)$ such that $xv \in E$ and $vy \notin E$ is called *lifting* if $\deg(x) \leq \deg(y)$ and *lowering* if $\deg(x) \geq 2 + \deg(y)$. A transformation ϕ of the graph G that replaces G with $\phi(G) = G - xv + vy$ is called an *edge rotation in the graph G about the vertex v corresponding to the triple of vertices* (x, v, y). For a lifting (lowering) triple (x, v, y), the corresponding edge rotation is called *lifting* (*lowering*). An edge rotation in a graph G is lifting if and only if its inverse is lowering in the graph $\phi(G)$. A bipartite graph $H = (V_1, E, V_2)$ is called a *bipartite threshold graph* if it has no lifting triples such that $x, y \in V_1$ and $v \in V_2$ or $x, y \in V_2$ and $v \in V_1$. The edge rotation corresponding to a triple of vertices (x, v, y) such that $x, y \in V_1$ and $v \in V_2$ ($x, y \in V_2$ and $v \in V_1$) is called a *V_1-rotation* (*V_2-rotation*) of edges. Every bipartite graph $H = (V_1, E, V_2)$ can be transformed to a bipartite threshold graph by a finite sequence of V_1-rotations (V_2-rotations) of edges. The aim of the paper is to give a polynomial algorithm that transforms every bipartite graph $H = (V_1, E, V_2)$ to a bipartite threshold graph by a shortest finite sequence of V_1-rotations of edges.

Keywords: algorithm, integer partition, threshold graph, bipartite graph, bipartite threshold graph, Ferrers diagram.

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Vitaly Anatol'evich Baransky, Dr. Phys.-Math. Sci., Prof., Ural Federal University, Yekaterinburg, 620000 Russia, e-mail: vitaly.baransky@urfu.ru .

Tatiana Aleksandrovna Senchonok, Cand. Phys.-Math. Sci., Prof., Ural Federal University, Yekaterinburg, 620000 Russia, e-mail: tatiana.senchonok@urfu.ru .

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