ON ONE GENERALIZED TRANSLATION AND THE CORRESPONDING INEQUALITY OF DIFFERENT METRICS

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In this paper, we discuss the properties of the generalized translation operator generated by the system of functions $\left\{\cos\left(\frac{(2k-1)\pi}{2}t\right)\right\}_{k=1}^{\infty}$, in the spaces $L^p(0,1)$, $p \geq 1$. The translation operator is applied to the study of Nikol'skii's inequality between the uniform norm and the L^p-norm of polynomials in this system.

Keywords: generalized translation operator, trigonometric polynomial, inequality of different metrics.

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