

MSC: 26A33 65H05

DOI: 10.21538/0134-4889-2022-28-4-273-276

ON THE FRACTIONAL NEWTON METHOD WITH CAPUTO DERIVATIVES

Emine Çelik, Yulong Li, Aleksey S. Telyakovskiy

Newton's method is commonly used to solve nonlinear algebraic equations due to its quadratic rate of convergence in the vicinity of the root. Multiple modifications of Newton's method are known, some lead to more stable calculations, although often at the expense of the rate of convergence. Here, derivative in Newton's method is replaced by Caputo fractional derivative, and the goal is to find all the roots, including complex, of nonlinear algebraic equation starting from the same real initial guess by varying the order of fractional derivative. This problem was analyzed by Akgül et al (2019), here some issues with their theoretical analysis and application of the method to the specific example are pointed out. The case of Caputo fractional derivatives of order $(0, 1]$ is analyzed. Akgül et al 2019 employ Caputo fractional Taylor's series of Odibat and Shawagfeh, 2007 for theoretical analysis. Specific issues with the paper are the following: 1) In iterative step integration in fractional derivative is done over interval $[\bar{x}, x_k]$, where \bar{x} is the unknown root, and x_k is the approximation of the root on the k -th iteration. 2) Expression for the derivative of fractional Taylor's series is only valid if derivative is evaluated over $[\bar{x}, x_k]$. 3) Expression for the rate of convergence is not correct. 4) In theoretical analysis, left fractional Caputo Taylor series is used, although if $x_{k+1} < \bar{x}$, then right fractional Taylor series should be used. 5) Numerical estimation of the rate of convergence gave value different from predicted by Akgül et al 2019. Plus, not clear over which interval integration was done to generate the numerical results.

Keywords: nonlinear equations, Caputo fractional derivative, Newton's method, convergence.

REFERENCES

1. Akgül A., Cordero A., Torregrosa J.R. A fractional Newton method with 2 α -th-order of convergence and its stability. *Appl. Math. Lett.*, 2019, vol. 98, pp. 344–351. doi: 10.1016/J.AML.2019.06.028.
2. Cao J.X., Li C.P., Chen Y.Q. High-order approximation to Caputo derivatives and Caputo-type advection-diffusion equations (II). *Fract. Calc. Appl. Anal.*, 2015, vol. 18, no. 3, pp. 735–761. doi: 10.1515/fca-2015-0045.
3. Kincaid D, Cheney W. *Numerical analysis: Mathematics of scientific computing, 3rd revised edition*. American Mathematical Society, 2002, 788 p. ISBN: 0821847880.
4. Li C.P. *High-order approximation to Caputo derivatives*. MATLAB Central File Exchange, 2021, <https://www.mathworks.com/matlabcentral/fileexchange/53924-high-order-approximation-to-caputo-derivatives> (Retrieved May 27, 2021).
5. Li C.P., Wu R.F., Ding H.F. High-order approximation to Caputo derivatives and Caputo-type advection-diffusion equations (I). *Communications in Appl. and Industr. Math.*, 2014, vol. 6, no. 2, e-536, pp. 1–32. doi: 10.1685/journal.caim.536.
6. Li H., Cao J.X., Li C.P. High-order approximation to Caputo derivatives and Caputo-type advection-diffusion equations (III). *J. of Comp. and Appl. Math.*, 2016, vol. 299, pp. 159–175. doi: 10.1016/j.cam.2015.11.037.
7. Odibat Z.M., Shawagfeh N.T. Generalized Taylor's formula. *Appl. Math. Comput.*, 2007, vol. 186, no. 1, pp. 286–293. doi: 10.1016/j.amc.2006.07.102.
8. Olsen J.S., Mortensen J., Telyakovskiy A.S. A two-sided fractional conservation of mass equation. *Advances in Water Resources*, 2016, vol. 91, pp. 117–121. doi: 10.1016/j.advwatres.2016.03.007.

Received July 4, 2022

Revised November 6, 2022

Accepted November 10, 2022

Emine Çelik, PhD, Lecturer, Department of Mathematics, Sakarya University, 54050, Sakarya, Türkiye, email: eminecelik@sakarya.edu.tr .

Yulong Li, PhD, Postdoc, Department of Mathematics & Statistics, University of Nevada, Reno, NV 89557, USA, email: yulongl@unr.edu; liyulong0807101@gmail.com .

Aleksey S. Telyakovskiy, PhD, Professor, Department of Mathematics & Statistics, University of Nevada, Reno, NV 89557, USA, email: alekseyt@unr.edu .

Cite this article as: Emine Çelik, Yulong Li, Aleksey S. Telyakovskiy. On the fractional Newton method with Caputo derivative. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 4, pp. 273–276 .