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INTERTWINING OF MAXIMA OF SUM OF TRANSLATES FUNCTIONS WITH NONSINGULAR KERNELS

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In previous papers we investigated so-called sum of translates functions $F(\mathbf{x},t) := J(t) + \sum_{j=1}^{n} \nu_j K(t-x_j)$, where $J : [0,1] \to \underline{\mathbb{R}} := \mathbb{R} \cup \{-\infty\}$ is a "sufficiently nondegenerate" and upper-bounded "field function", and $K : [-1,1] \to \underline{\mathbb{R}}$ is a fixed "kernel function", concave both on $(-1,0)$ and $(0,1)$, and also satisfying the singularity condition $K(0) = \lim_{t\to 0} K(t) = -\infty$. For node systems $\mathbf{x} := (x_1,\ldots,x_n)$ with $x_0 := 0 \le x_1 \le \cdots \le x_n \le 1 =: x_{n+1}$, we analyzed the behavior of the local maxima vector $\mathbf{m} := (m_0, m_1, \ldots, m_n)$, where $m_j := m_j(\mathbf{x}) := \sup_{x_j \le t \le x_{j+1}} F(\mathbf{x},t)$. Among other results we proved a strong intertwining property: if the kernel is decreasing on $(-1,0)$ and increasing on $(0,1)$, and the field function is upper semicontinuous, then for any two different node systems there are i,j such that $m_i(\mathbf{x}) < m_i(\mathbf{y})$ and $m_j(\mathbf{x}) > m_j(\mathbf{y})$. Here we partially succeed to extend this even to nonsingular kernels.

Keywords: minimax problems, kernel function, sum of translates function, vector of local maxima, equioscillation, intertwining of interval maxima.

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