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THE SPECTRUM OF ONE-DIMENSIONAL EIGENOSCILLATIONS OF TWO-PHASE LAYERED MEDIA WITH PERIODIC STRUCTURE

V. V. Shumilova

We study the spectrum of one-dimensional eigenoscillations along the Ox_1 axis of two-phase layered media with periodic structure occupying the band $0 < x_1 < L$. The period of the media is a band $0 < x_1 < \varepsilon$ composed of $2M$ alternating layers of an isotropic elastic or viscoelastic material (the first phase) and a viscous incompressible fluid (the second phase). It is assumed that the number of periods $N = L/\varepsilon$ is an integer, and the layers are parallel to the Ox_2x_3 plane. The spectrum is denoted by S_ε and is defined as the set of eigenvalues of a boundary value problem for a homogeneous system of ordinary differential equations with conjugation conditions at the interfaces between the solid and fluid layers. These conditions are derived directly from the initial assumption on the continuity of displacements and normal stresses at the interfaces between the layers. It is shown that the spectrum S_ε consists of the roots of transcendental equations, the number of which is equal to the number of periods N contained within the band $0 < x_1 < L$. The roots of these equations can only be found numerically, except for one particular case. In the case of multi-layered media with $N \gg 1$, the finite limits of the sequences $\lambda(\varepsilon) \in S_\varepsilon$ as $\varepsilon \to 0$ are proposed to be used as initial approximations. It is established that the set of all finite limits coincides with the set of roots of rational equations, denoted by S. The coefficients of these equations, and hence the points of the set S depend on the volume fraction of the fluid within the layered medium and do not depend on the number M of the fluid layers within the period. It is proved that for any $M \geq 1$ the spectrum S_ε converges in the sense of Hausdorff to the set S as $\varepsilon \to 0$. Keywords: spectrum of eigenoscillations, layered medium, two-phase medium, elastic material, viscoelastic material, viscous incompressible fluid.

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Vladlena Valerievna Shumilova, Dr. Phys.-Math. Sci., Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, 119526 Russia, e-mail: v.v.shumilova@mail.ru .

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