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## ON ESTIMATES OF LINEAR WIDTHS FOR CLASSES OF MULTIVARIATE FUNCTIONS IN THE LORENTZ SPACE

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We consider spaces of periodic multivariate functions, namely, the Lorentz space  $L_{p,\tau}(\mathbb{T}^m)$  and the Nikol'skii–Besov space  $S_{p,\tau,\theta}^r B$ , and study the order of linear widths of the class  $S_{p,\tau,\theta}^r B$ . The paper consists of the introduction and two sections. The introduction gives definitions, the notation used in the paper, and brief information on previous results on the issue under consideration. The first section contains two well-known statements that are often used in the proof of the main results. In the second section, order-exact estimates are established for the linear widths of the Nikol'skii–Besov class  $S_{p,\tau_1,\theta}^r B$  in the norm of the space  $L_{q,\tau_2}(\mathbb{T}^m)$  for different ratios of the parameters  $p, q, \tau_1, \tau_2$ , and  $\theta$ .

Keywords: linear widths, Lorentz space, the Nikol'skii–Besov class.

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