CONDITIONS UNDER WHICH THE SUMS OF ABSOLUTE VALUES OF BLOCKS IN THE FOURIER–WALSH SERIES OF FUNCTIONS OF BOUNDED VARIATION BELONG TO SPACES L^p

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In this paper, the following question is considered: under what conditions on a strictly increasing sequence of natural numbers $\{n_j\}_{j=1}^{\infty}$ does the sum of the series

$$\sum_{j=1}^{\infty} \left| \sum_{k=n_j}^{n_{j+1}-1} c_k(f) w_k(x) \right|,$$

where $c_k(f)$ are the Walsh–Fourier coefficients of a function f, belong to the space $L^p[0,1)$, $p > 1$, for any function f of bounded variation? For the case $p = \infty$, it is proved that such a sequence does not exist. For finite $p > 1$, sufficient conditions are obtained for the sequence $\{n_j\}$; these conditions are similar to the ones obtained by the first author in the trigonometric case.

Keywords: Walsh–Fourier series, functions of bounded variation, L^p-spaces.

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