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# ON A NEW CLASS OF TWO-DIMENSIONAL VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND WITH VARIABLE LIMITS OF INTEGRATION

**S. V. Solodusha**

The paper deals with linear two-dimensional Volterra integral equations of the first kind with variable lower and upper limits of integration. Such equations arise when describing the transient processes of a nonlinear dynamic system, represented as a finite segment (a polynomial) of the Volterra integro-power series. A new method for identifying symmetric kernels in the quadratic Volterra polynomial is presented, in which the input $x(t)$ and output $y(t)$ signals are scalar functions of time. The test signals used to solve this problem are chosen from the class of piecewise linear functions, which is explained by the specifics of the studied technical systems of the "input–output" type. This statement develops the approach based on test signals in the form of combinations of Heaviside functions and presented in the publications of A. S. Apartsyn. An explicit inversion formula is derived for a selected class of nonclassical Volterra equations of the first kind. Results about the existence and uniqueness of solutions of the corresponding integral equations are proved.

Keywords: nonlinear dynamic system, identification, Volterra equations.

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*Svetlana Vital'evna Solodusha,* Dr. Technic. Sci., Melentiev Energy Systems Institute of the Siberian Branch of the Russian Academy of Science; Matrosov Institute for System Dynamics and Control Theory of the Siberian Branch of the Russian Academy of Sciences, Irkutsk, 664033 Russia,
e-mail: solodusha@isem.irk.ru .

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