THE METHOD OF QUASI-SOLUTIONS BASED ON BARRIER FUNCTIONS IN THE ANALYSIS OF IMPROPER CONVEX PROGRAMMING PROBLEMS

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The paper is devoted to the construction of possible approximations for improper problems of convex programming based on the application of a classical approach to the regularization of ill-posed extremal problems, namely, V. K. Ivanov's method of quasi-solutions. While usually the constraints of the original problem in the method of quasi-solutions are aggregated with the help of exterior penalty functions, in this paper we use for this purpose a generalized inverse barrier function, which is a modification of interior penalty. Due to the specifics of the problem, we introduce a number of new control parameters into the minimized barrier function. Along with the penalty coefficients and the regularization parameter, we consider parameters that ensure the correctness of the application of the barrier method, first of all, the presence of interior points of the domain of the method. We also discuss the existence of solutions to emerging correction problems and analyze the influence of the parameters of the barrier function on the convergence of the proposed modification of the method of quasi-solutions for improper problems.

Keywords: convex programming, improper problem, optimal correction, method of quasi-solutions, barrier function methods.

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