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ON AN INTERPOLATION PROBLEM WITH THE SMALLESt L_2-NORM OF THE LAPLACE OPERATOR

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The paper is devoted to an interpolation problem for finite sets of real numbers bounded in the Euclidean norm. The interpolation is by a class of smooth functions of two variables with the minimum L_2-norm of the Laplace operator $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ applied to the interpolating functions. It is proved that if $N \geq 3$ and all interpolation points $\{(x_j, y_j)\}_{j=1}^N$ do not lie on the same line, then the minimum value of the L_2-norm of the Laplace operator on interpolants from the class of smooth functions for interpolated data from the unit ball of the space l_2^N is expressed in terms of the largest eigenvalue of the matrix of a certain quadratic form.

Keywords: interpolation, Laplace operator, thin plate splines.

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