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## A COMPLETE DESCRIPTION OF THE RELATIVE WIDTHS OF SOBOLEV CLASSES IN THE UNIFORM METRIC

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We consider the width of the Sobolev class of  $2\pi$ -periodic functions with  $\|f^{(r)}\|_\infty \leq 1$  with respect to the set of functions  $g$  such that  $\|g^{(r)}\|_\infty \leq M$  in the uniform metric  $K_n := K_n(W_\infty^r, MW_\infty^r, L_\infty)$ . We prove a lower bound on  $K_n$  for  $M = 1 + \varepsilon$  with small  $\varepsilon$ . This bound together with earlier results completes the analysis of the behaviour of  $K_n$ .

Keywords: Kolmogorov and relative widths.

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