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## ON THE STABILITY OF LINEAR TIME-VARYING DIFFERENTIAL EQUATIONS

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The article discusses the stability of linear differential equations with time-varying coefficients. It is shown that, in contrast to equations with time-invariant coefficients, the condition for the characteristic polynomial to be Hurwitz for a linear differential equation with time-varying coefficients is neither necessary nor sufficient for the asymptotic stability of the differential equation. It is proved that the analog of Kharitonov's theorem on robust stability does not hold if the coefficients of the differential equation are time-varying.

Keywords: linear differential equations, stability, time-varying system, stable polynomial, Kharitonov's theorem, robust stability.

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