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## ON THE SET OF NECESSARY OPTIMALITY CONDITIONS WITH POSITIONAL CONTROLS GENERATED BY WEAKLY DECREASING SOLUTIONS OF THE HAMILTON–JACOBI INEQUALITY

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Any weakly decreasing solution of the Hamilton–Jacobi inequality generates a so-called accessory problem of dynamic optimization over Krasovskii–Subbotin constructive motions (Euler curves) produced by extremal feedback control strategies. We derive conditions under which an optimal trajectory of the considered Mayer optimal control problem is a minimizer of the accessory problem for a fixed majorant — a certain solution of the Hamilton–Jacobi inequality. The result is formulated in terms of the compatibility of the latter solution with an optimal trajectory. In the general case of a nonsmooth majorant (and a nonsmooth problem), the optimality condition means that there is a component of the proximal subdifferential of the majorant along the optimal trajectory that coincides with a certain solution of an adjoint inclusion arising in the maximum principle of Kaskosz and Lojasiewicz. This implies the general feedback minimum principle — a global necessary optimality condition, which strengthens all known formulations of the maximum principle for problems without terminal constraints.

Keywords: extremals, feedback controls, weakly decreasing functions, feedback minimum principle.

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