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ANISOTROPY AND SPECTRAL ENTROPY: AXIOMATIC APPROACH

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Real-life dynamic systems operate under various disturbances and are affected by unknown external influences. That is why the problem of perturbation suppression is an extremely important branch of control theory. An effective approach to solving this problem is the anisotropic theory of stochastic robust control. Unfortunately, this theory has fundamental limitations—it is applicable only to discrete stochastic systems and only to stationary Gaussian sequences. Recently, attempts have been made to transfer the concepts of anisotropic theory to systems with continuous time. In this paper, the results of anisotropic theory are extended to arbitrary random signals, including both sequences with finite l_2 or power norm and sequences with arbitrary growth rate.

Keywords: linear systems, anisotropy, spectral entropy, σ -entropy norm.

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