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SOLUTION OF A LINEAR–QUADRATIC PROBLEM ON A SET OF PIECEWISE CONSTANT CONTROLS WITH PARAMETRIZATION OF THE FUNCTIONAL

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A linear–quadratic problem of optimal control with arbitrary matrices in the cost functional and a multidimensional control constrained at every time is considered. The set of admissible controls consists of piecewise constant vector functions relative to a nonuniform discretization grid. The optimal control problem is reduced to a finite-dimensional form with the use of characteristic functions with grid structure and block matrices together with the corresponding operation of scalar product. Nonnegative parameters of the quadratic forms provide the possibility of regularization of the cost functional. The choice of these parameters is aimed at the regularization of the functional in the sense of its reduction to a convex or concave structure at the level of a finite-dimensional model. The conditions for these parameters are of spectral nature; they are inequalities with respect to extreme eigenvalues of the block matrices that form the objective function. The corresponding convex or concave optimization problems allow to solve the problem in a finite number of iterations. A nongradient condition of global optimality is obtained for the original problem of optimal control based on known estimates for the increment of the functional. A nonlocal refinement procedure in terms of Pontryagin's function is proposed.

Keywords: linear–quadratic problem, multidimensional discrete control, functional with parameters, reduction to a finite-dimensional model, regularization of the problem.

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