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PROBLEMS OF BOUNDARY CONTROL AND OPTIMAL CONTROL OF STRING VIBRATIONS WITH MULTIPOINT INTERMEDIATE CONDITIONS ON THE STATE FUNCTIONS

V. R. Barseghyan

For the vibrating string equation with given initial and final conditions, the problems of boundary control and optimal control with given various multipoint intermediate conditions on the values of the deflection function and on the velocities of points of the string are considered. The control is performed both by displacement of one end with the other end fixed and by displacement at the two ends. The quality criterion is given for the whole time interval. Using the method of separation of variables, the problem is reduced to the problem of control and optimal control of ordinary differential equations with given initial, final, and unseparated multipoint intermediate conditions. For all problems according to a single scheme using methods of control theory of finite-dimensional systems with multipoint intermediate conditions, a constructive approach is proposed for finding functions of boundary control and optimal control of string vibrations that ensure the fulfillment of multipoint intermediate conditions.

Keywords: string vibrations, boundary control, vibration control, optimal control of vibrations, multipoint intermediate conditions.

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Vanya R. Barseghyan, Dr. Phys.-Math. Sci., Prof., Institute of Mechanics of National Academy of Sciences of RA; Yerevan State University, Yerevan, Armenia, e-mail: barseghyan@sci.am .

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